1. How many positive three-digit integers abc can represent a valid date in 2013, where either a corresponds to a month and bc corresponds to the day in that month or ab corresponds to a month and c corresponds to the day? For example, 202 is a valid representation for February 2nd, and 121 could represent either January 21st or December 1st.

2. Consider the numbers $\{24, 27, 55, 64, x\}$. Given that the mean of these five numbers is prime and the median is a multiple of 3, compute the sum of all possible positive integral values of x.

3. Nick has a terrible sleep schedule. He randomly picks a time between 4 AM and 6 AM to fall asleep, and wakes up at a random time between 11 AM and 1 PM of the same day. What is the probability that Nick gets at least 6 hours of sleep?

4. Given the digits 1 through 7, one can form $7! = 5040$ numbers by forming different permutations of the 7 digits (for example, 1234567 and 6321475 are two such permutations). If the 5040 numbers obtained are then placed in ascending order what is the 2013^{th} number?

5. An unfair coin lands heads with probability $\frac{1}{17}$ and tails with probability $\frac{16}{17}$. Matt flips the coin repeatedly until he flips at least one head and at least one tail. What is the expected number of times that Matt flips the coin?

6. A positive integer $b \geq 2$ is *neat* if and only if there exist positive base-b digits x and y (that is, x and y are integers and $0 < x, y < b$) such that the number $x.y$ base b (that is, $x + \frac{y}{b}$) is an integer multiple of x/y. Find the number of *neat* integers less than or equal to 100.

7. Robin is playing notes on an 88-key piano. He starts by playing middle C, which is the 40th lowest note on the piano. After playing a note, Robin plays with probability $\frac{1}{2}$ the lowest note that is higher than the note he just played, and with probability $\frac{1}{2}$ the highest note that is lower than the note he just played. What is the probability that he plays the highest note on the piano before playing the lowest note?

8. Big candles cost 16 cents and burn for exactly 16 minutes. Small candles cost 7 cents and burn for exactly 7 minutes. The candles burn at possibly varying and unknown rates, so it is impossible to predictably modify the amount of time for which a candle will burn except by burning it down for a known amount of time. Candles may be arbitrarily and instantly put out and relit. Compute the cost in cents of the cheapest set of big and small candles you need to measure exactly 1 minute.

9. Farmer John owns 2013 cows. Some cows are enemies of each other, and Farmer John wishes to divide them into as few groups as possible such that each cow has at most 3 enemies in her group. Each cow has at most 61 enemies. What is the minimal G such that, no matter which enemies they have, the cows can always be divided into at most G such groups?

10. Compute the number of positive integers b where $b \leq 2013$, $b \neq 17$, and $b \neq 18$ such that there exists some positive integer N such that $\frac{N}{17}$ is a perfect 17th power, $\frac{N}{18}$ is a perfect 18th power, and $\frac{N}{b}$ is a perfect bth power.