

1. What is  $\int_0^{10} (x-5) + (x-5)^2 + (x-5)^3 dx$ ?

2. Find the maximum value of

$$\int_{-\pi/2}^{3\pi/2} \sin(x)f(x) dx$$

subject to the constraint  $|f(x)| \leq 5$ .

3. Calculate

$$\int_{2^5}^{3^5} \frac{1}{x - x^{3/5}} dx.$$

4. Compute the  $x$ -coordinate of the point on the curve  $y = \sqrt{x}$  that is closest to the point  $(2, 1)$ .

5. Let

$$f(x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \frac{x^5}{5},$$

and set  $g(x) = f^{-1}(x)$ . Compute  $g^{(3)}(0)$ .

6. Compute

$$\lim_{x \rightarrow 0} \left( \frac{\sin x}{x} \right)^{\frac{1}{1 - \cos x}}.$$

7. A differentiable function  $g$  satisfies

$$\int_0^x (x-t+1)g(t) dt = x^4 + x^2$$

for all  $x \geq 0$ . Find  $g(t)$ .

8. Compute

$$\int_0^\infty \frac{\ln x}{x^2 + 4} dx.$$

9. Find the ordered pair  $(\alpha, \beta)$  with non-infinite  $\beta \neq 0$  such that  $\lim_{n \rightarrow \infty} \frac{\sqrt[n^2]{1!2! \cdots n!}}{n^\alpha} = \beta$  holds.

10. Find the maximum of

$$\int_0^1 f(x)^3 dx$$

given the constraints

$$-1 \leq f(x) \leq 1, \quad \int_0^1 f(x) dx = 0.$$