

1. Compute the minimum possible value of

$$(x-1)^2 + (x-2)^2 + (x-3)^2 + (x-4)^2 + (x-5)^2,$$

for real values of x .

2. Alice and Bob compete in Silly Math Tournament (SMT), in which a contestant's score is equal to the number of problems he or she gets right. The product of their scores is equal to three times the sum of their scores. Compute the sum of all possible scores for Bob.
3. Express $\frac{2^3-1}{2^3+1} \times \frac{3^3-1}{3^3+1} \times \frac{4^3-1}{4^3+1} \times \cdots \times \frac{16^3-1}{16^3+1}$ as a fraction in lowest terms.
4. The function $f(x)$ is known to be of the form $\prod_{i=1}^n f_i(a_i x)$, where a_i is a real number and $f_i(x)$ is either $\sin(x)$ or $\cos(x)$ for $i = 1, \dots, n$. Additionally, $f(x)$ is known to have zeros at every integer between 1 and 2012 (inclusive) except for one integer b . Find the sum of all possible values of b .
5. If f is a monic cubic polynomial with $f(0) = -64$, and all roots of f are non-negative real numbers, what is the largest possible value of $f(-1)$? (A polynomial is monic if it has a leading coefficient of 1.)
6. The quartic (4th-degree) polynomial $P(x)$ satisfies $P(1) = 0$ and attains its maximum value of 3 at both $x = 2$ and $x = 3$. Compute $P(5)$.
7. There exist two triplets of real numbers (a, b, c) such that $a - \frac{1}{b}$, $b - \frac{1}{c}$, and $c - \frac{1}{a}$ are the roots to the cubic equation $x^3 - 5x^2 - 15x + 3$, listed in increasing order. Denote those triplets as (a_1, b_1, c_1) and (a_2, b_2, c_2) . If a_1, b_1 , and c_1 are the roots to monic cubic polynomial f and a_2, b_2 , and c_2 are the roots to monic cubic polynomial g , find $f(0)^3 + g(0)^3$.
8. If x, y , and z are integers satisfying $xyz + 4(x + y + z) = 2(xy + xz + yz) + 7$, list all possibilities for the ordered triple (x, y, z) .
9. z_1 and z_2 are complex numbers that satisfy the equation $3z_1^2 - 2z_1z_2 + 2z_2^2 = 0$, and $\frac{z_1-2}{z_1+2}$ is a purely imaginary number, i.e. $\operatorname{Re}\left(\frac{z_1-2}{z_1+2}\right) = 0$. If P_1, P_2 , and O are points in the complex plane corresponding to z_1, z_2 , and 0, respectively, find the area of $\triangle P_1OP_2$.
10. Let $X_1, X_2, \dots, X_{2012}$ be chosen independently and uniformly at random from the interval $(0, 1]$. In other words, for each X_n , the probability that it is in the interval $(a, b]$ is $b - a$. Compute the probability that $\lceil \log_2 X_1 \rceil + \lceil \log_4 X_2 \rceil + \cdots + \lceil \log_{4024} X_{2012} \rceil$ is even. (Note: For any real number a , $\lceil a \rceil$ is defined as the smallest integer not less than a .)