1. A primary school on Mars is going to have a spring tour to Earth. They can rent three kinds of automatic space shuttles listed in the table below. If there are 8 martian teachers and 120 students on the trip what is the minimum amount of Martian dollars they would need to rent space shuttles in Martian dollars?

Shuttle Type	Passengers	Cost to Rent (Martian Dollars)
A	37	560
B	15	260
C	8	160

2. A dragon can eat a goat in 4 hours. A bear can eat a goat in 5 hours. A sloth can eat a goat in 100 hours. (Of course, they all prefer eating lobsters to eating goats.) If the dragon, bear, and sloth all start eating the same goat at the same time, how many hours does it take them to finish the goat? Assume they don't interfere with each other during their feast.

3. A bucket is currently full of water and has a mass of 29 kg. When it is half filled with water, it has a mass of 18.5 kg. How much mass will it have when it is filled to one third of maximum capacity?

4. Jordan has a paper that is a circle of radius 4. He wants to cut out a rectangle that is as large in area as possible. What is the largest possible area of a rectangle cut from this circle.

5. Alex pays the Papa Johns delivery man all 15 coins in his pocket, all of which are either pennies, nickles, dimes, quarters, or half dollars. He ends up paying the 74 cents. How many dimes did he pay with?

6. Let $f(n) = 1 + 2 + 3 + ... + n$ where n is an integer. What is the smallest n such that $f(n) > 100$?

7. What is the maximum number of points of intersection between a single variable polynomial of degree n and a single variable polynomial of degree m where $n \neq m$?

8. Given that $a_1 = 2$, $a_2 = 3$, $a_n = a_{n-1} + 2a_{n-2}$, what is $a_{100} + a_{99}$?

9. Let sequence A be $\{\frac{7}{4}, \frac{7}{6}, \frac{7}{9}, \ldots\}$ where the j^{th} term is given by $a_j = \frac{7}{4}\left(\frac{2}{3}\right)^{j-1}$. Let B be a sequence where the j^{th} term is defined by $b_j = a_j^2 + a_j$. What is the sum of all the terms in B?

10. Let $F(x)$ be a real-valued function defined for all real $x \neq 0, 1$ such that

$$F(x) + F\left(\frac{x-1}{x}\right) = 1 + x.$$

Find $F(2)$.

11. Stephen and Tal are now 2 km away from each other. They need to meet, so they are walking towards each other on a Main Street sidewalk. Stephen walks 1.2 m/s and Tal walks 1 m/s. They start walking at the same time. Stephen has a dog. When he starts, the dog starts running towards Tal at 2 m/s. When he meets Tal, he turns back and runs towards Stephen. He keeps on running at between Stephen and Tal until Stephen and Tal meet. How far will the dog run to the nearest tenth of a meter?

12. A fully charged laptop can operate for 286 minutes. When the battery is depleted, it takes 52 minutes to fully recharge it with the laptop turned off. How long would it take to fully charge the battery when it starts with 10% of the energy capacity and you are using the laptop at the same time.

13. Find the minimum of

$$\max\{a + b + c, b + c + d, c + d + e, d + e + f, e + f + g\}$$

where a, b, c, d, e, f, g are nonnegative reals with their sum 1.

14. Jim is given a dart board with a small circle that is worth 20 points and a ring surrounding the circle that is worth 11 points. No points are given if he does not hit any of these areas. What is the largest *integer* number of points that cannot be achieved with some combination of hits.

15. The sum of areas of rectangles with width $\frac{1}{n}$ and height $f\left(\frac{i}{n}\right)$ can be used to approximate the area between a curve and the x-axis on some interval. We wish to approximate the area of $f(x) = x^2 + x$ on the interval $0 \leq x \leq 1$. Find $\sum_{i=1}^{n} \frac{1}{n} f\left(\frac{i}{n}\right)$ in terms of n.

16. Let $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. In how many ways can two (not necessarily distinct) elements a, b be taken from S such that $\frac{a}{b}$ is in lowest terms, i.e. a and b share no common divisors other than 1?

17. Find all square numbers which can be represented in the form $2^a + 3^b$, where a, b are nonnegative integers. You can assume the fact that the equation $3^x - 2^y = 1$ has no integer solutions if $x \geq 3$.

18. A frog is jumping on the number line, starting at zero and jumping to seven. He can jump from x to either $x+1$ or $x+2$. However, the frog is easily confused, and before arriving at the number seven, he will turn around and jump in the wrong direction, jumping from x to $x-1$. This happens exactly once, and will happen in such a way that the frog will not land on a negative number. How many ways can the frog get to the number seven?

19. Call a nonnegative integer k *sparse* when all pairs of 1's in the binary representation of k are separated by at least two zeroes. For example, $9 = 1001_2$ is sparse, but $10 = 1010_2$ is not sparse. How many sparse numbers are less than 2^{17}?

20. Let $M = (-1, 2)$ and $N = (1, 4)$ be two points in the plane, and let P be a point moving along the x-axis. When $\angle MPN$ takes on its maximum value, what is the x-coordinate of P?

21. An *icosahedron* is a regular polyhedron with 12 vertices, 20 faces, and 30 edges. How many rigid rotations G are there for an icosahedron in $\mathbb{R}^3$?

22. Find the 2011th-smallest x, with $x > 1$, that satisfies the following relation:

$$\sin(\ln x) + 2\cos(3\ln x)\sin(2\ln x) = 0.$$

23. Marcus' pet ant is leashed up to the corner of a solid square brick with side length 1 unit. The length of the ant's leash is 6 units, and it can only travel on the ground and not through or on the brick. In terms of $x = \arctan\left(\frac{3}{4}\right)$, what is the area of region accessible to the ant?

24. Let ABC be any triangle, and D, E, F be points on $\overline{BC}$, $\overline{CA}$, $\overline{AB}$ such that $CD = 2BD$, $AE = 2CE$ and $BF = 2AF$. $\overline{AD}$ and $\overline{BE}$ intersect at X, $\overline{BE}$ and $\overline{CF}$ intersect at Y, and $\overline{CF}$ and $\overline{AD}$ intersect at Z. Find $\frac{Area(\triangle ABC)}{Area(\triangle XYZ)}$.

25. Find the maximum of

$$\frac{ab + bc + cd}{a^2 + b^2 + c^2 + d^2}$$

for reals a, b, c, and d not all zero.