1. Find $\int \frac{x+2}{(x-1)^2(x-2)}dx$.

2. Tangent lines are drawn at the points of inflection for the function $f(x) = \cos x$ on $[0, 2\pi]$. The lines intersect with the x-axis so as to form a triangle. What is the area of this triangle?

3. Let f be one of the solutions to the differential equation

$$f''(x) - 2xf'(x) - 2f(x) = 0.$$

Supposing that f has Taylor expansion

$$f(x) = 1 + x + ax^2 + bx^3 + cx^4 + dx^5 + \cdots$$

near the origin, find (a, b, c, d).

4. What is the value of the alternating harmonic series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots$?

5. Solve the integral equation

$$f(x) = \int_0^x e^{x-y} f'(y)dy - (x^2 - x + 1)e^x$$

for $f(x)$.

6. Evaluate the integral

$$\int_0^\pi |\sin(2x) - \sin(3x)|\ dx$$

and express your answer in the $\frac{a+b\sqrt{c}}{d}$, where a, b, c, and d are integers.

7. Let $f(x) = \frac{x^3 e^{(x^2)}}{1-x^2}$. Find $f^{(7)}(0)$, the 7th derivative of f evaluated at 0.

8. For the curve $\sin(x) + \sin(y) = 1$ lying on the first quadrant, find the constant α such that

$$\lim_{x\to 0} x^\alpha \frac{d^2y}{dx^2}$$

exists and is nonzero.

9. Evaluate the integral $\displaystyle\int_0^{\frac{\pi}{2}} \frac{dx}{1 + (\tan x)^\pi}$.

10. Evaluate the following integral:

$$\int_0^1 \frac{dx}{x(x+1)\left(\ln\left(1 + \frac{1}{x}\right)\right)^{2011}}$$