1. Sammy is a child math prodigy born on Jan 1. At age 0.1, he already excels at multiplication. At age 8, he decides to start saving money to attend Rice University, his top choice. He estimates that Rice's tuition plus room and board will cost \$45000 per year for each of the four years he will be in college. Being a clever investor of money, he can invest it in such a way that the annual interest rate is 100% compounded yearly (\$100 will earn \$100 in interest at the end of the first year, which will earn $100\% \times (100 + 100) = \$200$ in interest at the end of the next year). If Sammy starts to invest some of the cash he receives as birthday gift for his college fund every year at each of his birthdays, starting with his eighth and ending with his seventeenth (inclusive), what is the minimum *integer* amount of money he needs to deposit into the bank each year so that he will have enough money for Rice on his eighteenth birthday? Assume he invests the same amount of money every year (he receives sufficient cash as a birthday boy), and the interest applies at the end of each year. Also assume no inflation and constant interest rate.

2. Consider the curves $x^2 + y^2 = 1$ and $2x^2 + 2xy + y^2 - 2x - 2y = 0$. These curves intersect at two points, one of which is $(1, 0)$. Find the other one.

3. Find all rational roots of $|x - 1| \times |x^2 - 2| - 2 = 0$.

4. If r, s, t, and u denote the roots of the polynomial $f(x) = x^4 + 3x^3 + 3x + 2$, find

$$\frac{1}{r^2} + \frac{1}{s^2} + \frac{1}{t^2} + \frac{1}{u^2}$$

5. Compute

$$\sum_{n=1}^{\infty} \frac{(7n + 32) \cdot 3^n}{n \cdot (n + 2) \cdot 4^n}.$$

6. Find the remainder when $(x + 2)^{2011} - (x + 1)^{2011}$ is divided by $x^2 + x + 1$.

7. There are 2011 positive numbers with both their sum and their sum of reciprocals equal to 2012. Let x be one of these numbers. Find the maximum of $x + x^{-1}$.

8. Let $P(x)$ be a polynomial of degree 2011 such that $P(1) = 0$, $P(2) = 1$, $P(4) = 2$, ... , and $P(2^{2011}) = 2011$. Compute the coefficient of the x^1 term in $P(x)$.

9. It is a well-known fact that the sum of the first n k-th powers can be represented as a polynomial in n. Let $P_k(n)$ be such a polynomial for integers k and n. For example,

$$\sum_{i=1}^{n} i^2 = \frac{n(n + 1)(2n + 1)}{6},$$

so one has

$$P_2(x) = \frac{x(x + 1)(2x + 1)}{6} = \frac{1}{3}x^3 + \frac{1}{2}x^2 + \frac{1}{6}x.$$

Evaluate $P_7(-3) + P_6(-4)$.

10. How many polynomials P of degree 4 satisfy $P(x^2) = P(x)P(-x)$?