

1. Given 8 coins, at most one of them is counterfeit. A counterfeit coin is lighter than a real coin. You have a free weight balance. What is the minimum number of weighings necessary to determine the identity of the counterfeit coin if it exists?
2. Johnny Appleseed has many apple which contain apple seeds. Unfortunately, the ferocious deer often eats his apple. He wants to build the smallest rectangular fence for his apple farm, of size x by $x + 1$ such that the lengths add up to a prime number greater than 10. Also this prime number's digits add up to a prime number greater than 10. What will be the area of Johnny Appleseed's farm?
3. How many zeros are there at the end of $200!$?
4. What is the largest prime factor of 314159
5. A triangle has side lengths 7, 9, and 12. What is the area of the triangle?
6. Jeffrey went to Star Restaurant to order food for the math club. He has \$30 in total to spend. The menu has Kung Po Chicken, at \$8.00, egg rolls at \$0.60, and Won Ton Soup at \$2.00. Assume that he must spend the entire \$30, can only buy these three items. Also he must buy at least one of each of these three items. How many different combinations of these three items can he purchase? Assume no tax.
7. Find all solutions of $\frac{a}{x} = \frac{x-a}{a}$ for x .
8. A straight line connects City A at $(0,0)$ to City B, 300 meters away at $(300,0)$. At time $t=0$, a bullet train instantaneously sets out from City A to City B while another bullet train simultaneously leaves from City B to City A going on the same train track. Both trains are traveling at a constant speed of 50 meters/second. Also at $t=0$, a super fly stationed at $(150,0)$ meters and restricted to move on the same train tracks travels towards City B. The fly always travels at 60 meters/second, and any time it hits a train it instantaneously reverses its direction and travels at the same speed. At the moment the trains intersect, what is the total distance that the fly will have traveled? Assume the fly and each train is a point and that both the fly and the trains travel at their same respective velocities before and after collisions with each other.
9. Compute the base 10 value of 14641_n in terms of n for $n \geq 7$.
10. Frank has 60 meters of fence. What is the area of the largest regular hexagon that he can create?
11. A series of lockers, numbered 1 through 100, are all initially closed. Student 1 goes through and opens every locker. Student 2 goes through and closes every even locker. Student 3 goes through and "flips" every 3rd locker ("flipping" a locker means changing its state: if the locker is open he closes it, and if the locker is closed he opens it). Thus, Student 3 will close the third locker, open the sixth, close the ninth... This process continues with Students 1-100 going through and "flipping" every n^{th} locker. What is the number of the 7th open locker?
12. Episode 2: The Imperial Units Strike Back. Do you hate imperial units? Well imperial units hate you back! They have traveled many leagues – inch by inch, foot by foot, mile by mile. They have searched the fathoms and cables of the ocean for nautical miles on end, and acres of plains of grains. They would perch on rods and poles to stalk and attack you with this problem! If there are 500 corn plants per rood, 56 kernels per corn plant, and each kernel weighs exactly one grain, how many acres of corn plants are required to produce one long ton of food. The imperial units think that they have you beaten, but you know that there are 2240 pounds per long ton, 7000 grains per pound, and 4 roods per acre.
13. A wheel is rolled without slipping through 3 laps on a circular racecourse with radius 7. The wheel is perfectly circular and has radius 5. After the three laps, how many revolutions around its axis has the wheel been turned through?
14. An equilateral triangle is inscribed inside of a circle of radius R . Find the side length of the triangle.

15. Find the roots of $6x^4 + 17x^3 + 7x^2 - 8x - 4$
16. Given five circles of radii 1, 2, 3, 4, and 5, what is the maximum number of points of intersections possible (every distinct point where two circles intersect counts).
17. There are several alternatives to representation of numbers in positive radix positional numeral systems. For instance, negative integers can also be used as the radix in positional notation systems. For a given integer radix n , the digits $1, \dots, |n| - 1$ are used. Calculate the negadecimal (base -10) representation of 2010_{10} .
18. Evaluate $1 - \frac{2}{3} + \frac{4}{9} - \frac{8}{27} + \frac{16}{81} - \frac{32}{243}$
19. We need not restrict our number system radix to be an integer. Consider the phinary numeral system in which the radix is the golden ratio $\phi = \frac{1+\sqrt{5}}{2}$ and the digits 0 and 1 are used. Compute $1010100_\phi - .010101_\phi$ given that $F(n) = \frac{\phi^n - (-\phi)^{-n}}{\sqrt{5}}$ where $F(n)$ is the n th Fibonacci number ($F(0) = 0$, $F(1) = 1$, $F(n) = F(n-1) + F(n-2)$ for $n \geq 2$).
20. In the year 20102010 AD failing math class becomes a criminal offense. A student fails and is sentenced to a very long prison term. In order to keep track of time, he records the number of days that pass on a prison wall with a rock. However, since he failed math class, he can only write in unary (ie. with tally marks). He begins by making 1 mark on day 1, 2 marks on day 2, and n marks on day n . Each mark removes a piece of stone that is 1 in wide by 4 in tall by 1 in deep. The wall is 6 feet long by 8 feet wide by .5 foot deep. Additionally, he must dig a tunnel under a fence of volume 288 in^3 using the same method. On what day does the student escape by having removed the entire volume of the wall and dug the tunnel.