1. A duck and a mathematician walk into a bar. The duck didn't have any money, so she borrowed from the mathematician. After several non-alcoholic beverages, the drunk duck decides to try her luck and call a double or nothing. She mixes her two duck eggs with two chicken eggs and one ostrich egg.

   The mathematician is to pick an egg. If he picks a non-duck egg, then the duck is free from any money owed. Otherwise, she has to pay double. The 3 types of eggs are clearly distinctive, but because the mathematician is drunk, it is as if he picks at random. In addition, all the eggs have a $\frac{1}{3}$ chance of hatching. The duck, being a mother, doesn't want her eggs to hatch in a bar.

   What is the probability that she loses (pays double, or at least one of her duck eggs hatch)?

2. The quarter-imaginary positional number system has radix $2i$ and uses the digits $0, 1, 2,$ and 3. It can be used to represent any complex number without using a negative sign. For example, $1032_{2i} = 1(2i)^3 + 0(2i)^2 + 3(2i) + 1(2i)^0 = 1 - 2i$. Compute the base-10 representation of $3.\overline{0123}_{2i}$.

3. What is the smallest number of people that can be invited to The 2010 Rice Mathematics Tournament such that we are guaranteed either three of them met each other last year, or three of them did not meet each other last year?

4. Calculate $\cos^5\theta$ in terms of $\sum_{i=1}^{n} a_i \cos(k\theta)$ for some positive integers k and n and real numbers a_i.

5. Compute
$$\sum_{j=0}^{2010}\sum_{i=j}^{2010}\binom{i}{j}.$$

6. In an n-by-m grid, 1 row and 1 column are colored blue, the rest of the cells are white. If precisely $\frac{1}{2010}$ of the cells in the grid are blue, how many values are possible for the ordered pair (n, m)?

7. A bug either splits into two perfect copies of itself or dies. If the probability of splitting is $p > \frac{1}{2}$ (and is independent of the bug's ancestry), what is the probability that a bug's descendants die out? Express your answer as a function in terms of p.

8. Let $P(x)$ be a polynomial of degree n such that $P(k) = 3^k$ for $0 \le k \le n$. Find $P(n+1)$.

9. How many ordered pairs of complex numbers (x, y) satisfy
$$x^2 + y^2 = \frac{1}{x} + \frac{1}{y} = 9?$$

10. Compute the product of all integers such that $\lfloor \frac{n^2}{5} \rfloor$ is prime.