1. In the future, each country in the world produces its Olympic athletes via cloning and strict training programs. Therefore, in the finals of the 200 m free, there are two indistinguishable athletes from each of the four countries. How many ways are there to arrange them into eight lanes?

2. Factor completely the expression $(a-b)^3 + (b-c)^3 + (c-a)^3$.

3. If x and y are positive integers, and $x^4 + y^4 = 4721$, find all possible values of $x + y$.

4. How many ways are there to write 657 as a sum of powers of two where each power of two is used at most twice in the sum? For example, 256+256+128+16+1 is a valid sum.

5. Compute
$$\int_0^\infty t^5 e^{-t} dt$$

6. Rhombus ABCD has side length 1. The size of $\angle A$ (in degrees) is randomly selected from all real numbers between 0 and 90. Find the expected value of the area of ABCD.

7. An isosceles trapezoid has legs and shorter base of length 1. Find the maximum possible value of its area.

8. Simplify
$$\sum_{k=1}^{n} \frac{k^2(k-n)}{n^4}.$$

9. Find the shortest distance between the point (6,12) and the parabola given by the equation $x = \frac{y^2}{2}$.

10. Evaluate $\displaystyle\sum_{n=2009}^{\infty} \frac{\binom{n}{2009}}{2^n}$.

11. Let z_1 and z_2 be the zeros of the polynomial $f(x) = x^2 + 6x + 11$. Compute $(1 + z_1^2 z_2)(1 + z_1 z_2^2)$.

12. A number N has 2009 positive factors. What is the maximum number of positive factors that N^2 could have?

13. Find the remainder obtained when 17^{289} is divided by 7?

14. Let a and b be integer solutions to $17a + 6b = 13$. What is the smallest possible positive value for $a - b$?

15. What is the largest integer n for which $\frac{2008!}{31^n}$ is an integer?