

1. Let  $a$ ,  $b$ ,  $c$ , and  $d$  be the numbers that show when four fair dice, numbered 1 through 6 are rolled. What is the probability that  $|(a - 1)(b - 2)(c - 3)(d - 6)| = 1$ ?
2. Find all possibilities for the second-to-last digit of a number whose square is of the form  $1\_2.3.4.5.6.7.8.9.0$  (each  $\_$  is a digit).
3. Ten gears are lined up in a single file and meshed against each other such that the  $i^{\text{th}}$  gear from the left has  $5i + 2$  teeth. Gear  $i = 1$  (counting from the left) is rotated 21 times. How many revolutions does gear 10 make?
4. In the game Pokemawn, players pick a team of 6 different Pokemawn creatures. There are 25 distinct Pokemawn creatures, and each one belongs to exactly one of four categories: 7 Pokemawn are plant-type, 6 Pokemawn are bug-type, 4 Pokemawn are rock-type, and 8 Pokemawn are bovine-type. However, some Pokemawn do not get along with each other when placed on the same team: bug-type Pokemawn will eat plant-type Pokemawn, plant-type Pokemawn will eat rock-type Pokemawn, and bovine-type Pokemawn will eat anything except other Bovines. How many ways are there to form a team of 6 different Pokemawn such that none of the Pokemawn on the team want to eat any of the other Pokemawn?
5. Four cards are drawn from a standard deck (52 cards) with suits indistinguishable (for example,  $A\spadesuit$  is the same as  $A\clubsuit$ ). How many distinct hands can one obtain?
6. Find all complex numbers  $z$  such that  $z^5 = 16\bar{z}$ , where if  $z = a + bi$ , then  $\bar{z} = a - bi$ .
7. Evaluate  $\sqrt{\frac{1+\sqrt{3}i}{2}}$
8. Frank alternates between flipping a weighted coin that has a  $\frac{2}{3}$  chance of landing heads and a  $\frac{1}{3}$  chance of landing tails and another weighted coin that has a  $\frac{1}{4}$  chance of landing heads and a  $\frac{3}{4}$  chance of landing tails. The first coin tossed is the “ $2/3 - 1/3$ ” weighted coin. What is the probability that he sees two heads in a row before he sees two tails in a row?
9. The triangular numbers  $T_n = 1, 3, 6, 10, \dots$  are defined by  $T_1 = 1$  and  $T_{n+1} = T_n + (n + 1)$ . The square numbers  $S_n = 1, 4, 9, 16, \dots$  are defined by  $S_1 = 1$  and  $S_{n+1} = T_{n+1} + T_n$ . The pentagonal numbers  $P_n = 1, 5, 12, 22, \dots$  are defined by  $P_1 = 1$  and  $P_{n+1} = S_{n+1} + T_n$ . What is the 20th pentagonal number  $P_{20}$ ?
10. Evaluate  $e^{i\pi/3} + 2e^{2i\pi/3} + 2e^{3i\pi/3} + 2e^{4i\pi/3} + e^{5i\pi/3} + 9e^{6i\pi/3}$ .