1. **Answer: 2**

   The exponent evaluates to 0; $5^0 = 1$ so the least integer greater is 2.

2. **Answer: 15**

   The primes less than 50 are: $2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47$.

3. **Answer: $\sqrt{2}$**

   The expression factors into $\left(x-\sqrt{2}\right)\left(x+\sqrt{2}\right)(x+2)$. Thus, $\sqrt{2}$ is the only positive root.

4. **Answer: 330**

   The only Pythagorean triplet containing 11 is $(11, 60, 61)$, giving a triangle with area $\frac{11 \cdot 60}{2}$.

5. **Answer: 3**

   The conversion formula $C = (F-32) \cdot \frac{5}{9}$ requires that $F - 32$ be a multiple of 9 for C to be an integer. Such integers between 45 and 70 are 50, 59, and 68.

6. **Answer: $\frac{11\pi}{96}$**

   The volume is $\pi \cdot 7 \cdot \left(\frac{1}{8}\right)^2 + \frac{1}{3}\pi \cdot 1 \cdot \left(\frac{1}{8}\right)^2 = \pi\left(\frac{7}{64} + \frac{1}{192}\right)$.

7. **Answer: $\frac{32}{35}$**

   From the information provided, we can see that 64% of contestants are both earing blue jeans and tennis shoes. Therefore, $\frac{64}{70} = \frac{32}{35}$ of the contestants wearing tennis shoes are wearing blue jeans.

8. **Answer: $\frac{3}{8}$**

   If his n^{th} move is rock, his $(n+1)^{th}$ move will never be rock. If his n^{th} move is not rock, then his $(n+1)^{th}$ move will be rock with probability $\frac{1}{2}$. So his second move will always be non-rock. His third move will be rock with probability $\frac{1}{2}$. His fourth move can only be rock if his third is not, so this happens with probability $\frac{1}{4}$. His fifth move can be rock only if his fourth is not, so the probability of this happening is $\frac{3}{4} \cdot \frac{1}{2} = \frac{3}{8}$.

9. **Answer: 77**

   The product $1 \cdot 2 \cdot 3 \cdots 19 \cdot 20$ contains all primes from 2 to 19, so the sum is $2+3+5+7+11+13+17+19 = 77$.

10. **Answer: $\frac{5}{324}$**

    The probability of player n winning if no one has already done so is $\frac{n}{6}$. Restated, the probability that player n will NOT win if no one has already done so is $\frac{6-n}{6}$. So the probability that the first 5 players will all lose is $\frac{5}{6} \cdot \frac{4}{6} \cdot \frac{3}{6} \cdot \frac{2}{6} \cdot \frac{1}{6} = \frac{5}{324}$, and this is the only way the sixth player can win.

11. **Answer: $\frac{3+\sqrt{7}}{2}$**

$$\sqrt[3]{\frac{17+\sqrt{7}+45}{4}} = \sqrt[3]{\frac{34\sqrt{7}+90}{8}} = \frac{\sqrt[3]{(3+\sqrt{7})^3}}{2} = \frac{3+\sqrt{7}}{2}$$

12. **Answer: 98°**

    Let O be the center of the circle. Since tangents and radii are at right angles, $m\angle AOB = 360° - 90° - 90° - 16° = 164°$. The measure of the arc intercepted by $\angle ACB$ is then $360° - 164° = 196$, so $m\angle ACB = 196/2$

13. **Answer: 3225600**

    This very long word contains 34 letters. One of them (I) is repeated seven times, four (S, C, A, L) are repeated three times, five (U, P, E, R, O) are repeated twice, and the other five (F, G, T, X, D) occur once. So $N = \frac{34!}{(2!)^5(3!)^4 7!} = 2^{19}3^9 5^6 7^3 11^3 13^2 17^2 \cdot 19 \cdot 23 \cdot 29 \cdot 31$, and the number of factors of N is $(19+1)(9+1)(6+1)(3+1)(3+1)(2+1)(2+1)(1+1)(1+1)(1+1) = 3225600$.

14. **Answer:** $\mathbf{\frac{27}{128}}$

The ways to have four grandchildren are: two children each with two children, two children with one and three children, and three children with one, one, and two children. There are two orders for the second case and three for the third, so the total probability is:

$$\left(\frac{1}{2}\right)^3 + 2 \cdot \frac{1}{2} \cdot \left(\frac{1}{4}\right)^3 + 3 \cdot \left(\frac{1}{4}\right)^3 \cdot \frac{1}{2}$$

15. **Answer: 4.5 or** $\mathbf{\frac{9}{2}}$

At time t, you have run $10t$ meters, and the raptor has run $15\sqrt{2}(t-3)$ meters. Since the raptor is only running north and east, it has to run $\sqrt{2}$ times as far as you, so it catches you when $\sqrt{2} \cdot 10t = 15\sqrt{2}(t-3)$, or $10t = 30(t-3)$. This gives $20t = 90$, or $t = \frac{9}{2} = 4.5$.

16. **Answer:** $\mathbf{\frac{3\sqrt{3}}{4}}$

Since the hexagon is convex, a 120-degree rotation around P must send H to X, X to G, and G to H. But that means that $\triangle GHX$ is equilateral with center P! If $PX = 1$, the triangle must have a median length of $\frac{3}{2}$, and thus a side length of $\sqrt{3}$. So its area is $\frac{3\sqrt{3}}{4}$.

17. **Answer: 150093**

Let $\circ$ denote the four operations. For parentheses to be used, the expression must be of the form $(n) \circ m$ or $n \circ (m)$, giving $2 \cdot 4 \cdot 9^2$ possibilities. For two operations to be used, the expression must be of the form $l \circ m \circ n$, giving $9^3 \cdot 4^2$ possibilities. For one operation to be used, the form must be $k \circ lmn$, $kl \circ mn$, or $klm \circ n$, giving $3 \cdot 9^4 \cdot 4$ possibilities. Finally, there are 9^5 five-digit numbers. Adding up gives the answer.

18. **Answer:** $\mathbf{\frac{79}{288}}$

Note that eating 3, 4, or 5 tacos requires 9, 14, or 20 mL of hot sauce, respectively. If x, y, z represent the numbers of 3-taco, 4-taco, and 5-taco days, then emptying the bottle requires $9x + 14y + 20z \geq 88$, which has three solutions in the integers: $(0, 2, 3), (0, 1, 4), (0, 0, 5)$. The total probability of these three cases is

$$\binom{5}{2}\left(\frac{1}{3}\right)^2\left(\frac{1}{2}\right)^3 + \binom{5}{1}\left(\frac{1}{3}\right)\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^5 = 10 \cdot \frac{1}{9} \cdot \frac{1}{8} + 5 \cdot \frac{1}{3} \cdot \frac{1}{16} + \frac{1}{32}$$

19. **Answer: 2007**

Notice that if only the third and fourth pirate remain, the third pirate can enact his proposal simply by voting for it himself. Thus, he would be able to get all of the gold pieces, giving none to the fourth pirate.

Therefore, if only the second, third, and fourth pirates remain, the second pirate can save 2007 gold pieces for himself and offer 0 to the third pirate and just 1 to the fourth pirate. Notice that the fourth pirate will support this proposal, since otherwise the second pirate would walk the plank, leaving the above case in which the fourth pirate gets no gold at all.

So, when the first pirate proposes his offer, he need only offer the third pirate a single gold piece, and save the rest for himself. Analogously, the third pirate will support this proposal, because otherwise the second pirate's proposal would leave him with nothing. Therefore, the highest-ranking pirate ends up with 2007 gold pieces.

20. **Answer: 14**

The smallest sum of three distinct squares is $1 + 4 + 9 = 14$, which conveniently can be expressed as $14 = 2 \cdot 7$ and $14 = 2 + 5 + 7$.

21. **Answer: 2**

    Pick $x = \sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\cdots}}}}$. Then by substitution $x = \sqrt{2+x}$; squaring both sides gives $x^2 = 2 + x$. Since we assumed that all square roots in the definition for x refer to the positive values, x is the positive root of the equation $x^2 - x - 2 = 0$. Hence,

    $$x = \frac{1+\sqrt{1+8}}{2} = 2.$$

22. **Answer: 20**

    The most possible shared faces is 5, so the surface area is that of the five cubes minus two for each shared face, or $6 \cdot 5 - 2 \cdot 5 = 20$.

23. **Answer: $\frac{1}{3}$**

    Obviously, the closer your direction of travel is to the direction of B, the better. Let P be a point such that P is 1000 feet from A and exactly 1000 feet from B. There are two such points, and they are mirror images of each other across $\overline{AB}$, so let the other one be P'. Since $\triangle PAB$ is equilateral, $m\angle PAB = 60°$. Likewise, $m\angle P'AB = 60°$, so $m\angle PAP' = 120°$. If we walk in a direction that takes us between P and P', we will end up less than 1000 feet from B. The probability of this happening is $\frac{120}{360} = \frac{1}{3}$.

24. **Answer: $\sqrt{2}$**

    Let the triangle be $\triangle ABC$ with hypotenuse $\overline{BC}$, and let P be the midpoint of the hypotenuse, which is also the center of each semicircle. Clearly, the outer semicircle has radius $\frac{\sqrt{2}}{2}$. Now let X be the point of tangency of the inner circle to $\overline{AB}$. Then $\overline{PX} \perp \overline{AB}$. But then $\triangle XBP \sim \triangle ABC$, so $PX = \frac{1}{2}BC = \frac{1}{2}$. Since PX is a radius of the smaller circle, the ratio of the radii is thus $\sqrt{2}$.

25. **Answer: $1 + \frac{2\sqrt{3}}{3}$**

    As seen in the figure, the centers of the unit circles form an equilateral triangle of side length 2. Therefore an altitude of the triangle is $\sqrt{3}$ so $OB = \frac{2}{3}\sqrt{3}$ and the radius $OA = 1 + \frac{2\sqrt{3}}{3}$.