Geometry Test
2007 Rice Math Tournament
February 24, 2007

1. An equilateral triangle has perimeter numerically equal to its area, which is not zero. Find its side length.

2. Two spheres of radius 2 pass through each other's center. Find the surface area of the regular octahedron inscribed within the space enclosed by both spheres.

3. Cumulation of a polyhedron means replacing each face with a pyramid of height h using the face as a base. There is a cumulation of the cube of side length s which (after removing unecessary edges) has twelve sides, each a congruent rhombus. What is the height h used in this cumulation?

4. Nathan is standing on vertex A of triangle ABC, with $AB = 3$, $BC = 5$, and $CA = 4$. Nathan walks according to the following plan: He moves along the altitude-to-the-hypotenuse until he reaches the hypotenuse. He has now cut the original triangle into two triangles; he now walks along the altitude to the hypotenuse of the larger one. He repeats this process forever. What is the total distance that Nathan walks?

5. Given an octahedron with every edge of length s, what is the radius of the largest sphere that will fit in this octahedron?

6. Let $RICE$ be a quadrilateral with $IE = 8$, $IC = 4$, $m\angle R = 30°$, $m\angle CER = 60°$, and $m\angle RIE = m\angle ICE$. Find CE.

7. Two congruent, regular tetrahedra of edge length 2 are positioned such that their heights (vertex to center of base) coincide but the tetrahedra themselves do not. Find the volume of the region in which they overlap.

8. ΔABC has $\overline{AB} = \overline{AC}$. Points M and N are midpoints of $\overline{AB}$ and $\overline{AC}$, respectively. The medians $\overline{MC}$ and $\overline{NB}$ intersect at a right angle. Find $(\frac{\overline{AB}}{\overline{BC}})^2$.

9. Points P, Q, R, S, T lie in the pane with S on $\overline{PR}$ and R on $\overline{QT}$. If $PQ = 5$, $PS = 3$, $PR = 5$, $QS = 3$, and $RT = 4/\sqrt{3}$, what is ST?

10. A car starts moving at constant speed at the origin facing in the positive y-direction. Its minimum turning radius is such that it the soonest it can return to the x-axis is after driving a distance d. Let Γ be the boundary of the region the car can reach by driving at most a distance d; find an $x > 0$ so that $\left(x, \frac{d}{3} + \frac{d\sqrt{3}}{2\pi}\right)$ is on Γ.