# General Test
# 2007 Rice Math Tournament
# February 24, 2007

1. There are three bins: one with 30 apples, one with 30 oranges, and one with 15 of each. Each is labeled "apples," "oranges," or "mixed." Given that all three labels are wrong, how many pieces of fruit must you look at to determine the correct labels?

2. Aliens from Lumix have one head and four legs, while those from Obscra have two heads and only one leg. If 60 aliens attend a joint Lumix and Obscra interworld conference, and there are 129 legs present, how many heads are there?

3. Mary puts one red and one blue marble into a box. In another box she places two red marbles. She then forgets which box is which and randomly reaches into one of the boxes and takes out a red marble. What is the probability that the other marble in that box is blue?

4. Evaluate $(\tan 10°)(\tan 20°)(\tan 30°)(\tan 40°)(\tan 50°)(\tan 60°)(\tan 70°)(\tan 80°)$.

5. Two disks of radius 1 are drawn so that each disk's circumference passes through the center of the other disk. What is the circumference of the region in which they overlap?

6. Team Rice has a $\frac{1}{3}$ chance of winning any given math contest. If Rice competes in 4 contests this semester, what is the probability that the team will win at least once?

7. A boat is traveling upstream at 5 mph relative to the current flowing against it at 1 mph. If a tree branch 10 miles upstream from the boat falls into the current of the river, how many hours does it take to reach the boat?

8. Tina writes four letters to her friends Silas, Jessica, Katie, and Lekan. She prepares an envelope for Silas, an envelope for Jessica, an envelope for Katie, and an envelope for Lekan. However, she puts each letter into a random envelope. What is the probability that no one receives the letter they are supposed to receive?

9. Peter Pan and Crocodile are each getting hired for a job. Peter wants to get paid 6.4 dollars daily, but Crocodile demands to be paid 10 cents on day 1, 20 cents on day 2, 40 cents on day 3, 80 cents on day 4, and so on. After how many whole days will Crocodile's total earnings exceed that of Peter's?

10. Al, Betty and Clara are in the same class of 50 students total, but are not friends with each other. Al is friends with 24 students, Betty is friends with 39, and Clara is friends with 20. What is the greatest number of students that could be friends with all 3 of them?

11. Jonathan finds out that his ideal match is Sara Lark, but to improve his odds of finding a girlfriend, he is willing to date any girl whose name is an anagram of "Sara Lark", and whose name consists of both a first and last name of at least one letter. How many such anagrams are there?

12. Pete has some trouble slicing a 20-inch (diameter) pizza. His first two cuts (from center to circumference of the pizza) make a 30° slice. He continues making cuts until he has gone around the whole pizza, each time trying to copy the angle of the previous slice but in fact adding 2° each time. That is, he makes adjacent slices of 30°, 32°, 34°, and so on. What is the area of the smallest slice?

13. A rope of length 10 $m$ is tied tautly from the top of a flagpole to the ground 6 $m$ away from the base of the pole. An ant crawls up the rope and its shadow moves at a rate of 30 $cm/min$. How many meters above the ground is the ant after 5 minutes? (This takes place on the summer solstice on the Tropic of Cancer so that the sun is directly overhead.)

14. Let there be 50 natural numbers $a_i$ such that $0 < a_1 < a_2 < \cdots < a_{50} < 150$. What is the greatest possible sum of the differences $d_j$, where each $d_j = a_{j+1} - a_j$?

15. A number $x$ is uniformly chosen on the interval $[0, 1]$, and $y$ is uniformly randomly chosen on $[-1, 1]$. Find the probability that $x > y$.

16. Find the area of a square inscribed in an equilateral triangle, with one edge of the square on an edge of the triangle, if the side length of the triangle is $2 + \sqrt{3}$.

17. There is a test for the dangerous bifurcation virus that is 99% accurate. In other words, if someone has the virus, there is a 99% chance that the test will be positive, and if someone does not have it, then there is a 99% chance the test will be negative. Assume that exactly 1% of the general population has the virus. Given an individual that has tested positive from this test, what is the probability that he or she actually has the disease? Express your answer as a percentage.

18. A farmer wants to build a rectangular region, using a river as one side and some fencing as the other three sides. He has 1200 feet of fence which he can arrange to different dimensions. He creates the rectangular region with length $L$ and width $W$ to enclose the greatest area. Find $L + W$.

19. Arrange the following four numbers from smallest to largest: $a = (10^{100})^{10}$, $b = 10^{(10^{10})}$, $c = 1000000!$, $d = (100!)^{10}$

20. Let there be $4n + 2$ distinct paths in space with exactly $2n^2 + 6n + 1$ points at which exactly two of the paths intersect. (A path never intersects itself.) What is the maximum number of points where exactly three paths intersect?

21. Convert the following decimal to a common fraction in lowest terms: $0.92007200720072007\cdots$ (or $0.9\overline{2007}$).

22. Katie begins juggling five balls. After every second elapses, there is a chance she will drop a ball. If she is currently juggling $k$ balls, this probability is $\frac{k}{10}$. Find the expected number of seconds until she has dropped all the balls.

23. A quadrilateral has side lengths 3, 3, $x$, and $y$, where $x$ and $y$ are integers. We are allowed to choose $x$ arbitrarily, then we choose $y$. Let $N$ be the number of possible integer values for $y$ after $x$ is chosen. Find the greatest number of possible values for $N$.

24. Andy and Bob are playing a ping-pong match. Right now, Andy has 17 points and Bob has 18 points. On any particular play, Andy has a 50% chance of winning, which gains him 1 point. If Andy loses, Bob gains 1 point. The game ends when one player wins by getting 21 points. What is the probability that Andy will win?

25. Nathan is standing on vertex $A$ of triangle $ABC$, with $AB = 3$, $BC = 5$, and $CA = 4$. Nathan walks according to the following plan: He moves along the altitude-to-the-hypotenuse until he reaches the hypotenuse. He has now cut the original triangle into two triangles; he now walks along the altitude to the hypotenuse of the larger one. He repeats this process forever. What is the total distance that Nathan walks?