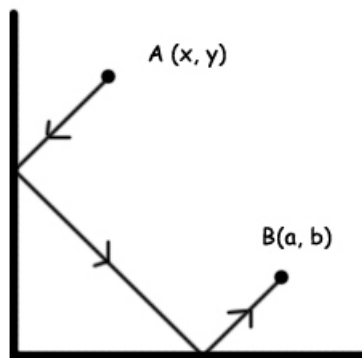


GENERAL TEST
2006 RICE MATH TOURNAMENT
FEBRUARY 25, 2006

1. After a cyclist has gone $\frac{2}{3}$ of his route, he gets a flat tire. Finishing on foot, he spends twice as long walking as he did riding. How many times as fast does he ride as walk?
2. A customer enters a supermarket. The probability that the customer buys (a) bread is .60, (b) milk is .50 and (c) both bread and milk is .30. What is the probability that the customer would buy either bread or milk or both?
3. After a typist has written ten letters and had addressed the ten corresponding envelopes, a careless mailing clerk inserted the letters in the envelopes at random, one letter per envelope. What is the probability that **exactly** nine letters were inserted in the proper envelopes?
4. In a certain tournament bracket, a player must be defeated three times to be eliminated. If 512 contestants enter the tournament, what is the greatest number of games that could be played?
5. A geometric sequence is one where the ratio between each two consecutive terms is constant (for example, 3, 6, 12, 24, . . .). The fifth term of a geometric series is 5!, and the sixth term is 6!. What is the fourth term?
6. An alarm clock runs 4 minutes slow every hour. It was set right $3\frac{1}{2}$ hours ago. Now another clock which is correct shows noon. In how many minutes, to the nearest minute, will the alarm clock show noon?
7. An aircraft is equipped with three engines that operate independently. The probability of an engine failure is .01. What is the probability of a successful flight if only one engine is needed for the successful operation of the aircraft?
8. Given two 2's, "plus" can be changed to "times" without changing the result: $2+2 = 2\cdot 2$. The solution with three numbers is easy too: $1+2+3 = 1\cdot 2\cdot 3$. There are three answers for the five-number case. Which five numbers with this property has the largest sum?
9. If to the numerator and denominator of the fraction $\frac{1}{3}$ you add its denominator 3, the fraction will double. Find a fraction which will triple when its denominator is added to its numerator and to its denominator and find one that will quadruple.
10. What is the square root of the sum of the first 2006 positive odd integers?
11. An insurance company believes that people can be divided into 2 classes: those who are accident prone and those who are not. Their statistics show that an accident prone person will have an accident in a yearly period with probability 0.4, whereas this probability is 0.2 for the other kind. Given that 30% of people are accident prone, what is the probability that a new policyholder will have an accident within a year of purchasing a policy?
12. What is the largest prime factor of 8091?
13. $123456789 = 100$. Here is the only way to insert 7 pluses and/or minus signs between the digits on the left side to make the equation correct: $1 + 2 + 3 - 4 + 5 + 6 + 78 + 9 = 100$. Do this with only three plus or minus signs.
14. Determine the area of the region defined by $x^2 + y^2 \leq \pi^2$ and $y \geq \sin x$.

15. The odometer of a family car shows 15,951 miles. The driver noticed that this number is palindromic: it reads the same backward as forwards. "Curious," the driver said to himself, "it will be a long time before that happens again." Surprised, he saw his third palindromic odometer reading (not counting 15,951) exactly five hours later. How many miles per hour was the car traveling in those 5 hours (assuming speed was constant)?
16. Points A_1, A_2, \dots are placed on a circle with center O such that $\angle OA_n A_{n+1} = 35^\circ$ and $A_n \neq A_{n+2}$ for all positive integers n . What is the smallest $n > 1$ for which $A_n = A_1$?
17. Car A is traveling at 20 miles per hour. Car B is 1 mile behind, following at 30 miles per hour. A fast fly can move at 40 miles per hour. The fly begins on the front bumper of car B, and flies back and forth between the two cars. How many miles will the fly travel before it is crushed in the collision?
18. Alex and Brian take turns shooting free throws until they each shoot twice. Alex and Brian have 80% and 60% chances of making their free throws, respectively. What is the probability that after each free throw they take, Alex has made at least as many free throws as Brian if Brian shoots first?
19. When the celebrated German mathematician Karl Gauss (1777-1855) was nine years old, he was asked to add all the integers from 1 through 100. He quickly added 1 and 100, 2 and 99, and so on for 50 pairs of numbers each adding in 101. His answer was $50 \cdot 101 = 5,050$. Now find the sum of all the digits in the integers from 1 through 1,000,000 (i.e. all the digits in those numbers, not the numbers themselves).
20. Given a random string of 33 bits (0 or 1), how many (they can overlap) occurrences of two consecutive 0's would you expect? (i.e. "100101" has 1 occurrence, "0001" has 2 occurrences)
21. How many positive integers less than 2005 are relatively prime to 1001?
22. A certain college student had the night of February 23 to work on a chemistry problem set and a math problem set (both due on February 24, 2006). If the student worked on his problem sets in the math library, the probability of him finishing his math problem set that night is 95% and the probability of him finishing his chemistry problem set that night is 75%. If the student worked on his problem sets in the chemistry library, the probability of him finishing his chemistry problem set that night is 90% and the probability of him finishing his math problem set that night is 80%. Since he had no bicycle, he could only work in one of the libraries on February 23rd. He works in the math library with a probability of 60%. Given that he finished both problem sets that night, what is the probability that he worked on the problem sets in the math library?
23. Consider two mirrors placed at a right angle to each other and two points A at (x, y) and B at (a, b) . Suppose a person standing at point A shines a laser pointer so that it hits both mirrors and then hits a person standing at point B (as shown in the picture). What is the total distance that the light ray travels, in terms of $a, b, x,$ and y ? Assume that $x, y, a,$ and b are positive.



24. The number 555,555,555,555 factors into eight distinct prime factors, each with a multiplicity of 1. What are the three largest prime factors of 555,555,555,555?
25. For positive integers n let $D(n)$ denote the set of positive integers that divide n and let $S(n) = \sum_{k \in D(n)} \frac{1}{k}$. What is $S(2006)$? Answer with a fraction reduced to lowest terms.