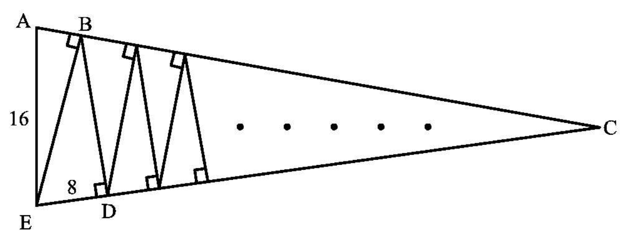
General Test
2005 Rice Math Tournament
February 26, 2005

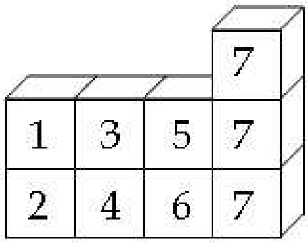
1. Trevor's farm of mutant animals has 3 legged goats and 5 legged goats. In one pen he counts 83 legs and 23 heads, how many 5 legged goats are there?
2. A point in the x-y plane is called a lattice point if its coordinates are both integers. What is maximum possible area of a circle in the plane that does not contain any lattice points in its interior?
3. A city is circled by a city wall. There are two straight roads that run from the wall to the center of the city. The distance between the intersections of the roads with the wall is the same walking along the roads as walking along the top of the wall. At what angle (in radians) do the roads meet?
4. Using 1, 2, and 3 to make 4-digit positive integers, how many include both 1 and 2?
5. In the game of math ball, teams can score by making a basket which yields 13 points or kicking a field goal which yields 7 points. What is the largest integer that is not a possible score for a team to attain?
6. David W. Leebron's first grandchild will be born on December 8, 2019. What day of the week will that be?
7. If I roll two 20-sided dice and a 4-sided die, what is the probability that the sum will be 21?
8. If 2005! is written in Base 26, how many 0's does the number end in?
9. At William Rice's Marsh, there are an infinite number of magic lillypads numbered 1, 2, 3, and so on. A magic lillypad lights up if a frog jumps on it while it is not lit, and turns off if a frog jumps on it while it is lit. Suppose all lillipads are initially turned off. Conor the frog begins by hopping on the first lillypad and then continues hopping on every lillypad thereafter. Conor's friend Bob starts hopping after Conor and begins by hopping on the second pad and continues by hopping on the fourth, sixth, eighth, and so on. Shortly after Bob, Dan hops on the third, sixth, ninth, and so on lillypads. If there is a frog for each positive number n that hops on every n^{th} pad, what is the number on the m^{th} lillypad that remains lit in the end?
10. A quadrilateral in the plane is formed so that for every vertex (x, y), x and y are integers and $x^2+y^2 = 50$. How many such quadrilaterals are parallelograms?
11. in how many (different) ways can the words in this problem be ordered (including the words in parentheses)?
12. A dog is tied via a 30 ft. leash to one corner of a 10 ft. by 20 ft. dog pen. Given that the dog is initially on the outside of the pen and that neither he (nor his leash) can cross the pen's fence, what area of land does he have to roam in? Answer in terms of π.
13. Given 4 consecutive positive integers, where n is the lowest, what is the smallest n for which the product of these four numbers is divisible by 128?
14. An infinite series of similar right triangles converges to point C. If $\overline{AE} = 16$, and $\overline{ED} = 8$, what is the sum of all the vertical segments $(\overline{AE} + \overline{BD} + ...)$?



15. You roll a fair 12-sided die and a fair 8-sided die. What is the probability that the sum is divisible by 5?

16. Which is bigger, the coefficient of x^{1000} in $(1-x^2+x^5)^{2005}$ or the coefficient of x^{1000} in $(1+x^2-x^5)^{2005}$?

17. There are 9 blocks. Three are labelled with the number 7 and the others are labelled from 1 to 6. A boy builds the formation below by placing the blocks one at a time while reading the number on each block as it is placed. How many different sequences of 9 integers could have been read?



18. How many of the first one million positive integers are integer powers of 4 or 6?

19. A ball rolls back and forth at a constant $25\frac{ft}{sec}$ between 2 walls 100ft apart. The walls begin closing toward each other at $20\frac{ft}{sec}$ each. Assume the ball starts at one wall and starts rolling the instant the walls begin moving. How far has it traveled once the walls close together trapping the ball?

20. How many perfect cubes are there in the sequence $1^1, 2^2, 3^3, 4^4, ..., 100^{100}$?

21. You have five containers in front of you of sizes 30, 42, 70, 105, and 210 mL. If you use a subset of these containers to measure 1 mL, and n is the sum of their volumes, in mL, compute the smallest possible volume of n.

22. Find the area enclosed by the graph of $|x+y|+|x|+|y|=1$.

23. If the roots of x^3+ax^2+bx+c are three consecutive positive integers, then what are all possible values of $\frac{a^2}{b+1}$?

24. A sequence of letters rolls off the tongue if the following two conditions are met:
1) The sequence does not begin or end with two consecutive consonants.
2) No three consecutive letters are all consonants.
How many permutations of "MATHEMATICS" roll off the tongue?

25. Given a random string of 33 bits (0 or 1), how many (they can overlap) occurrences of two consecutive 0's would you expect? (i.e. "100101" has 1 occurrence, "0001" has 2 occurrences)