

ALGEBRA TEST  
2005 RICE MATH TOURNAMENT  
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1. On a test, the average score for the girls in the class is 91, and the average score for the boys in the class is 85. If the average score for the class is 89, what fraction of the class are boys?
2. How many distinct real roots does the following equation have?

$$x^4 + 8x^2 + 16 = 4x^2 - 12x + 9$$

3. At William Rice's Marsh, there are an infinite number of magic lily pads numbered 1, 2, 3, and so on. A magic lily pad lights up if a frog jumps on it while it is not lit, and turns off if a frog jumps on it while it is lit. Suppose all lily pads are initially turned off. Conor the frog begins by hopping on the first lily pad and then continues hopping on every lily pad thereafter. Conor's friend Bob starts hopping after Conor and begins by hopping on the second pad and continues by hopping on the fourth, sixth, eighth, and so on. Shortly after Bob, Dan hops on the third, sixth, ninth, and so on lily pads. If there is a frog for each positive number  $n$  that hops on every  $n^{\text{th}}$  pad, what is the number on the  $m^{\text{th}}$  lily pad that remains lit in the end?
4. Ashley Ann Allen, a hapless algebra student, sees the expression  $\frac{\log A}{\log B}$ . She mistakenly cancels the "log"s, to get the expression  $\frac{A}{B}$ . Miraculously, when she plugs in values for A and B, she gets the correct answer. Assuming  $A \neq B$ , find all possible ordered pairs (A,B)
5. Let  $f(x)$  and  $g(x)$  be functions which take integers as arguments. Let

$$f(x + y) = f(x) + g(y) + 8$$

for all integers  $x$  and  $y$ . Let  $f(x) = x$  for all negative numbers  $x$ , and let  $g(8) = 17$ . What is  $f(0)$ ?

6. Let

$$x = \left\lfloor \frac{2007 \cdot 2006 \cdot 2004 \cdot 2003}{\frac{1}{3} \cdot 2005^4} \right\rfloor,$$

where  $\lfloor x \rfloor$  is the greatest integer less than or equal to  $x$ . Find  $(x^2 + 1) \left( ((x^2 + 1) \cdot x^2)^2 + 1 \right)$ .

7. If the roots of  $x^3 + ax^2 + bx + c$  are three consecutive positive integers, then what are all possible values of  $\frac{a^2}{b+1}$ ?
8. Find all ordered pairs of digits  $(a, b)$  such that the 6 digit number  $24ab32$  is divisible by 99.
9. If  $a, b, c$  are real numbers such that

$$a + b + c = -1$$

$$a^2 + b^2 + c^2 = 17$$

$$a^3 + b^3 + c^3 = 11$$

Find  $abc$ .

10. A monic polynomial is one in which the coefficient of the highest order term is 1. Find the monic polynomial  $p(x)$  (with integer coefficients) of least degree that satisfies  $p(\sqrt{2} + \sqrt{5}) = 0$ .