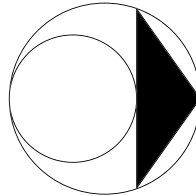
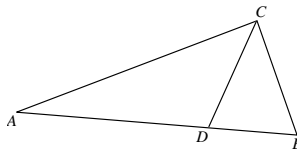


GEOMETRY TEST  
 2004 RICE MATH TOURNAMENT  
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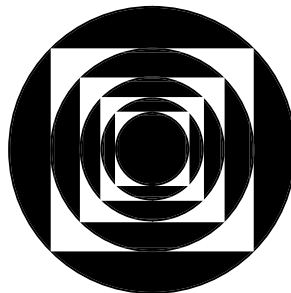
1. In the diagram below, the outer circle has radius 3, and the inner circle has radius 2. What is the area of shaded region?



2. A parallelogram has 3 of its vertices at  $(1, 2)$ ,  $(3,8)$ , and  $(4, 1)$ . Compute the sum of the possible  $x$ -coordinates for the 4th vertex.
3.  $AC$  is 2004.  $CD$  bisects angle  $C$ . If the perimeter of  $ABC$  is 6012, find  $\frac{AC \times BC}{AD \times BD}$ .



4.  $P$  is inside rectangle  $ABCD$ .  $PA = 2$ ,  $PB = 3$ , and  $PC = 10$ . Find  $PD$ .
5. Find the area of the region of the  $xy$ -plane defined by the inequality  $|x| + |y| + |x + y| \leq 1$ .
6. We inscribe a square in a circle of radius 1 and shade the region between them. Then we inscribe another circle in the square and another square in the new circle and shade the region between the new circle and square. After we have repeated this process infinitely many times, what is the area of the shaded region?



7. Yet another trapezoid  $ABCD$  has  $AD$  parallel to  $BC$ .  $AC$  and  $BD$  intersect at  $P$ . If  $\frac{[ADP]}{[BCP]} = \frac{1}{2}$ , find  $\frac{[ADP]}{[ABCD]}$ . (Here the notation  $[P_1 \cdots P_n]$  denotes the area of the polygon  $P_1 \cdots P_n$ .)
8. A triangle has side lengths 18, 24, and 30. Find the area of the triangle whose vertices are the incenter, circumcenter, and centroid of the original triangle.

9. Given is a regular tetrahedron of volume 1. We obtain a second regular tetrahedron by reflecting the given one through its center. What is the volume of their intersection?
10. Right triangle  $XYZ$  has right angle at  $Y$  and  $XY = 228$ ,  $YZ = 2004$ . Angle  $Y$  is trisected, and the angle trisectors intersect  $XZ$  at  $P$  and  $Q$  so that  $X, P, Q, Z$  lie on  $XZ$  in that order. Find the value of  $(PY + YZ)(QY + XY)$ .