Electricity Demand Analysis Using Cointegration and Error-Correction Models with Time Varying Parameters: The Mexican Case [1]

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Abstract

We specify and estimate a double-log functional form of the demand equation, using monthly Mexican electricity data for residential, commercial and industrial sectors. Income, prices and a nonparametric temperature measure are used as explanatory variables, and the income elasticity is allowed to evolve slowly over time by employing the time varying coefficient (TVC) cointegrating model. The specification of the proposed TVC cointegrating model is justified by testing it against the spurious regression and the usual fixed coefficient (FC) cointegratin regression. The estimated coefficients suggest that the income elasticity has followed a predominantly increasing path for all sectors during the entire sample period, and that electricity prices do not significantly affect in the longrun the residential and commercial demand for electricity in Mexico.

First Draft: June 4, 2001
This version: July 23, 2003

JEL Classification: C51, C52, C53, Q41.
Key words and phrases: Electricity demand, time varying coefficients, cointegration, canonical cointegrating regression, error-correction model, and series estimation.

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1. Introduction

Cointegration analysis and error correction models (ECM) have become the standard techniques for the study of electricity demand since their formal development by Engle and Granger (1987) and their early application to the forecast of the electricity demand in Engle et al. (1989). Subsequent developments related to this approach have relied on the use of new techniques to identify cointegrating relationships (for example, the Johansen's method (1988, 1991)), as well as on the inclusion of more specific energy-related variables in the model. Some recent examples of these extensions include the work by Beenstock, et al. (1999) which analyzes the demand for electricity in Israel, and the one by Silk and Joutz (1997) which constructs an appliance stock index to study US residential electricity demand. In most of these kinds of analyses the demand equation is specified as a linear double-log function, as a way to obtain elasticities directly from its coefficients, and the parameters are estimated using data whose time span is rather long, going beyond forty years in some cases.

Despite the relative popularity of the above techniques, the long time span covered by these studies raises serious concerns about the validity of the fixed coefficients (FC) assumption in the electricity demand equation. This assumption in a double-log functional form of demand simply implies constant elasticities for the entire sample period under study. This feature of the model is indeed questionable in light of the changes that could have taken place in the economy over such a long period of time affecting the demand for electricity. See Hass and Schipper (1998) for more discussions on this issue. Specific examples of the determinants for such changes include the efficiency improvements in electrical equipments, the developmental stage of the economy (whether the economy is in transition to later stages of development and industrialization), and even the government energy policy and the habit persistence of consumers. These determinants are not static, but rather tend to evolve slowly through time and thus they constantly modify the responses of the aggregated electricity demand to variations in income and prices. Therefore, if we use data collected over a relatively long time period to estimate an electricity demand function, we should at least consider the possibility that the parameters in the regression may not be constant.

As a way to capture this evolving nature of the electricity demand, some studies have employed alternative functional forms for the demand equations, such as linear functions, to indirectly calculate the elasticities as functions of the current levels of the variables and the parameters of the demand equation (see, for example, Chang and Hsing (1991)). However, these studies fail to see the possibility of allowing the relevant parameters to vary over time. Another alternative suggested in the literature is to introduce some structural changes into the model, but this approach has obvious shortcomings. Among others, it can neither handle the dynamics of the parameter changes nor provide the perspective of their possible future evolution. To our knowledge, these alternatives have been applied in the energy literature without cointegration analysis, which is necessary to properly identify the longrun relationships among the relevant variables (see, for example, Hass and Schipper (1998)).

In this paper, we estimate the electricity demand function for Mexico by applying a new alternative model that combines cointegration analysis with time varying coefficients in the demand equation. In other words, we look for cointegrating relationships in the electricity market that change over time. Park and Hahn (1999) have developed the idea of the time varying coefficients cointegrating regression, which we employ here to estimate the time varying coefficient (TVC) on the income variable in a double-log functional form of the electricity demand. The underlying assumption in the model is that the TVC is generated by a smooth function that can be approximated by a series of functions which expands appropriately as the sample size grows. Taking into account the intrinsically slow changes in the general factors that affect electricity demand, such as technology, developmental stage of the economy and habit persistency, this basic assumption seems plausible. Park and Hahn (1999) demonstrated that under some regularity conditions the estimators of general TVC cointegration models are consistent, efficient and asymptotically Gaussian. Thus, in addition to the advantage of allowing for flexible coefficients, this approach exploits the available information efficiently to estimate the parameters of the model, and gives a valid basis for forecasting the possible future path of the TVC.

To highlight the essence of the TVC cointegrating model we use a basic electricity demand equation with electricity prices and income as explanatory variables. In particular we focus our analyses on the models with time varying income elasticities for three Mexican sectors: residential, commercial and industrial, and work with the data that span fifteen years from 1985 to 2000. As a proxy for the income variable, we use private consumption for the residential and commercial sectors and industrial production for the industrial sector. As we use monthly data to estimate longrun relationships, we need to properly model the seasonality present in the electricity demand data. For this, we construct a seasonal variable from what we call the temperature response function, which is estimated nonparametrically using intraday temperature data, such as the measures taken every three hours. This seasonal variable provides a temperature measure that reflects the variations in electricity demand due to the factors related to the changes in temperature.

The resulting demand equation is then estimated by the canonical cointegrating regression (CCR) method developed in Park (1992) to effectively deal with the endogeneity and the serial dependence of the error introduced as we approximate TVC by a series of functions. The validity of the proposed TVC cointegration model is examined by performing the specification tests against the spurious regression with TVC and also against the usual FC cointegration model, using Wald-type variable addition tests. The results support the cointegration against spuriousness and TVC against the FC specification. The presence of TVC in the cointegrating relationship also allows for more flexible formulation of ECM which may include more than one lags of the error correction term. The usual ECM based on a FC cointegration model is limited to include only one lagged error correction term due to the collinearity with the lags of the error correction terms and the lagged differences of the other included variables. Finally, we conduct an out-of-sample forecast exercise over a twelve-month forecast horizon to evaluate the performance of our proposed TVC cointegration model, compared to those of the FC cointegration model and the ECM derived from our TVC cointegration model. Based on the root mean square error criterion, the TVC cointegration performed better than the FC cointegration model for all three sectors. On the other hand, the ECM performed better than the TVC cointegration model for the residential and commercial sectors, which was expected since the forecast horizon is relatively short. In the industrial sector, however, the ECM was outperformed by the TVC cointegration model, indicating that the industrial electricity demand is more responsive to long term changes than to temporal shocks. This may be explained by the rigidities and the constraints present in the energy market, such as the inflexibilities of the industrial equipments to switch among different energy sources and the limited infrastructure for the distribution of alternative energy sources.

The rest of the paper is organized as follows. Section 2 introduces the theoretical background for the TVC cointegrating regression model, the CCR methodology, and the model for the seasonality of electricity demand along with the temperature response function. In Section 3, we present the empirical results from the estimation of the temperature response function, the TVC cointegrating regression, the ECM derived from the TVC cointegration model and the FC cointegration model. Also provided are the out-of-sample forecast comparison among the three models for each sector. Finally, some concluding remarks are provided in Section 4.

2. The Model

2.1 The Time Varying Cointegrating Regression Model

The prototype electricity demand model used in the literature takes the form

$$d_t = \pi + \gamma y_t + \delta p_t + \phi z_t + u_t,$$

where d_t denotes the demand for electricity, y_t the income or production, p_t the real price of electricity, z_t the variable that captures the seasonal component of the demand,[2] u_t the stationary error and π, γ, δ and ϕ the parameters to be estimated. All of the economic variables are expressed in natural logarithms.

[2] A detailed description on how to construct this seasonal variable z_t is given in Section 2.3.

Let $x_t = (y_t, p_t, z_t)'$, and $\bar{\alpha} = (\gamma, \delta, \phi)'$. Then we may rewrite the above model as

$$d_t = \pi + \bar{\alpha}' x_t + u_t. \tag{1}$$

In this linear double-log functional form of the demand, the parameters in the vector $\bar{\alpha}$ represents the elasticities, and they are assumed to be constant over the entire sample period under study. However there is a possibility that the longrun relationships among the variables change through time, especially when we are analyzing the model over a relatively long time period. In order to take into account the time varying nature of the elasticities, we may allow the parameters to evolve over time and accordingly specify the model as

$$d_t = \pi + \alpha_t' x_t + u_t, \tag{2}$$

where the coefficients α_t are now allowed to change over time.

Furthermore, given that the responses of the electricity demand to the changes in the exogenous variables could be affected by slowly evolving factors such as the degree of economic development or the habit persistence of the agents, it is assumed that α_t changes in a smooth way. Specifically we let

$$\alpha_t = \alpha\left(\frac{t}{n}\right), \tag{3}$$

where n is the sample size, $t \in \{0, 1, 2, ..., n\}$ and α is a smooth function defined on the unit interval [0,1]. Note that the subscript indicating the dependence of α_t on n has been suppressed for notational simplicity. As an estimand of the function α, we define the functional

$$\Pi(\alpha) = (\alpha(r_1), ..., \alpha(r_d))', \tag{4}$$

where r_i is a number in the interval [0,1].

If the function α in (3) is sufficiently smooth, then it is well known that α can be approximated arbitrarily well by a linear combination of a sufficiently large number of polynomial and/or trigonometric functions on [0,1]. For our study, we consider the Fourier Flexible Form (FFF) functions, which include a constant, a linear function, $\varphi(r) = r$, and k pairs of trigonometric series functions, $(\varphi_i(r))_{i=1}^k$, where each pair $\varphi_i(r)$ is defined as $\varphi_i(r) = (\cos \lambda_i r, \sin \lambda_i r)'$ with $\lambda_i = 2\pi i$. That is, we assume that the smooth function α can be approximated by the function α_k defined as

$$\alpha_k(r) = \beta_{k,1} + \beta_{k,2}\, r + \sum_{i=1}^{k} (\beta_{k,2i+1}, \beta_{k,2(i+1)})\, \varphi_i(r), \tag{5}$$

with $\beta_{k,j} \in R^3$ for $j = 1, 2, \ldots, 2(k+1)$ and for some k. In fact, Park and Hahn (1999) showed that the function α given in (3) can be arbitrarily well approximated by (α_k) by increasing the number k of the included trigonometric pairs.[3] Alternatively, if we define $f_k(r) = (1, r, \varphi_1'(r), ..., \varphi_k'(r))'$ with $r \in [0,1]$ and $\beta_k = (\beta_{k,1}', \beta_{k,2}', ..., \beta_{k,2(k+1)}')'$, the function α_k defined in (5) can be rewritten as

$$\alpha_k = (f_k' \otimes I_3)\beta_k, \tag{6}$$

and the estimand $\Pi(\alpha_k)$ as

$$\Pi(\alpha_k) = (\alpha_k(r_1)', ..., \alpha_k(r_d)')' = T_k\beta_k, \tag{7}$$

where T_k is the matrix given by $T_k = F_k \otimes I_3$ with $F_k = (f_k(r_1), ..., f_k(r_d))'$ for $k \geq 1$.

Estimation of α_k and $\Pi(\alpha_k)$ involves the estimation of the parameters in β_k, as can be seen from (6) and (7). A natural way to estimate these parameters is to apply the ordinary least squares (OLS) to the regression

$$d_t = \pi + \beta_k' x_{kt} + u_{kt}, \tag{8}$$

where

$$\begin{aligned} x_{kt} &= f_k\left(\frac{t}{n}\right) \otimes x_t, \\ u_{kt} &= u_t + (\alpha - \alpha_k)\left(\frac{t}{n}\right)' x_{kt}, \end{aligned}$$

using the notations defined earlier. Note that the new error u_{kt} includes an additional term representing the error from approximating the original smooth function α in (3) by the series function α_k introduced in (5). From the OLS estimators $\hat{\beta}_{nk} = (\hat{\beta}_{nk,1}', \hat{\beta}_{nk,2}', \ldots, \hat{\beta}_{nk,2(k+1)}')'$ of β_k from regression (8), the sample estimates of α_k and $\Pi(\alpha_k)$ can be easily obtained by substituting β_k with its sample estimate $\hat{\beta}_{nk}$ in (6) and (7). They are given by $\hat{\alpha}_{nk} = (f_k' \otimes I_3)\hat{\beta}_{nk}$ and $\Pi(\hat{\alpha}_{nk}) = T_k\hat{\beta}_{nk}$, where the subscript n is used to make it explicitly that the parameter estimates depend on the sample.

Park and Hahn (1999) showed that $\Pi(\hat{\alpha}_{nk})$ is a consistent estimator of $\Pi(\alpha)$ if the number k of the trigonometric pairs included in the series (5) increases along with the sample size n at an appropriate rate. The required expansion rate for k is determined by the smoothness of the function α and the moment conditions of the underlying time series. The required rate becomes slower as the function becomes smoother or as the number of the existing moments of the underlying time series gets smaller.[4] Of course, the conditions on the smoothness of the function α cannot be verified, since α is not observable, and therefore the validity of the

[3] This result holds if it is assumed that α is at least twice differentiable, with bounded derivatives on [0,1]. See Park and Hahn (1999, Lemma 2).

[4] The explicit assumption is that $k = cn^r$ with $2/(2q-1) < r < (p-2)/3p$, where c is a constant, n is the sample size, p is the number of moments of the underlying variables and q the number of derivatives of α. See Park and Hahn (1999, Assumption 4).

resulting estimators is based on our perception of the way the time varying coefficients evolve through time. They also showed that the convergence rate of the series estimators is $n^{-1}k$, which is slower, by a factor of k, than the convergence rate of the OLS estimators for the usual FC models.

Due to the endogeneity of the error term, the OLS estimators of the TVC cointegration model (8) are asymptotically inefficient, and in general non-Gaussian,[5] which invalidates the standard OLS-based inferential procedures. To obtain efficient estimators and a valid inferential basis for the parameters in our TVC model (8), we employ, as in Park and Hahn (1999), the canonical cointegrating regression (CCR) method developed by Park (1992). The CCR method is based on the transformations of the variables that are correlated in the longrun with the error term, which effectively remove the longrun endogeneity and the serial correlation effects in the errors. In the following section, we layout the CCR procedure for the estimation of our TVC cointegrating model (2) or (8).

2.2 CCR Estimation

For the estimation the TVC cointegrating model (8) by the CCR method, we first construct the required transformations for the variables $d_{t,}$ y_t and p_t using their stationary components. Let $\tilde{x}_t = (y_t, p_t)'$, $v_t = \Delta\tilde{x}_t$ and $w_t = (u_t, v_t')'$, where (u_t) are the stationary errors in the original TVC model (2). For the process w_t, we also need to define the long run covariance matrix $\Omega = \sum_{k=-\infty}^{\infty} \mathbf{E} w_t w_{t-k}'$, the contemporaneous covariance matrix $\Sigma = \mathbf{E} w_0 w_0'$, and the one-sided longrun covariance matrix $\Delta = \sum_{k=0}^{\infty} \mathbf{E} w_t w_{t-k}'$. We partition Ω, Σ and Δ conformably with the partition of w_t into cell submatrices Ω_{ij}, Σ_{ij} and Δ_{ij}, for $i, j = 1, 2$. Note that Σ_{11} and Ω_{11} represent, respectively, the short and longrun variances of the error u_t.

The CCR estimation of the TVC cointegrating model (8) is based on the regression

$$d_t^* = \pi + \beta_k' x_{kt}^* + u_{kt}^*, \tag{9}$$

whose elements are defined by

$$\begin{aligned} d_t^* &= d_t - \left(f_k\left(\frac{t}{n}\right) \otimes \Delta_2 \Sigma^{-1} w_t \right)' \beta_k - (0, \Omega_{12}\Omega_{22}^{-1}) w_t, \\ x_{kt}^* &= \left(f_k\left(\frac{t}{n}\right) \otimes \tilde{x}_t^*,\ z_t \right), \\ u_{kt}^* &= u_t^* + (\alpha - \alpha_k)\left(\frac{t}{n}\right) x_{kt}, \end{aligned}$$

using the transformed nonstationary explanatory variables $\tilde{x}_t^*$ and the modified error u_t^* given below

$$\tilde{x}_t^* = \tilde{x}_t - \Delta_2 \Sigma^{-1} w_t, \quad u_t^* = u_t - \Omega_{12}\Omega_{22}^{-1} \Delta\tilde{x}_t,$$

[5] See Park and Hahn (1999, Theorem 7).

where $\Delta_2 = (\Delta_{12}', \Delta_{22}')$. We note that the longrun variance of the CCR error (u_t^*) is given by

$$\varpi_*^2 = \Omega_{11} - \Omega_{12}\Omega_{22}^{-1}\Omega_{21}, \tag{10}$$

which is the conditional longrun variance of the error (u_t) given the innovations (v_t) of the regressors $(y_t, p_t)'$, and is strictly less than the longrun variance Ω_{11} of u_t, unless the regressors are strictly exogeneous. Hence the CCR estimation, i.e., the OLS estimation of the CCR transformed model (9), yields efficient and optimal estimators. In practice, non-parametric methods can be employed to compute consistent estimates of Ω and Δ using the fitted residuals from the OLS estimation of model (8).[6] Denote by $\hat{\pi}^*$ and $\hat{\beta}_{nk}^*$ the CCR estimators, which are the OLS estimators of model (9). Then we may use them to obtain the efficient estimators $\hat{\alpha}_{nk}^* = (f_k' \otimes I_3)\hat{\beta}_{nk}^*$ and $\Pi(\hat{\alpha}_{nk}^*) = T_k\hat{\beta}_{nk}^*$ for α_k and $\Pi(\alpha)$ from the relationships given in (6) and (7). Park and Hahn (1999) demonstrated that the CCR estimator $\Pi(\hat{\alpha}_{nk}^*)$ is a consistent estimator of $\Pi(\alpha)$ and that its limit distribution is normal.[7]

The consistency and efficiency of the CCR estimators of the TVC cointegrating model (9) are attained presuming that the original time varying coefficient model (2) or (8) is correctly specified. Hence it remains to justify the adequacy of the model (2), and we will do so by performing two specification tests proposed in Park and Hahn (1999), which are Wald-type variable addition tests introduced originally in Park (1990). The first statistic τ^* tests whether or not the TVC model (2) is cointegrated against the alternative that the model is spurious. Specifically, the test is defined as

$$\tau^* = \frac{RSS_{TVC} - RSS_{TVC}^s}{\varpi_*^2}, \tag{13}$$

where ϖ_*^2 is the long run variance estimate of (u_t^*) given in (10), and RSS_{TVC} and RSS_{TVC}^s are the sums of squared residuals, respectively, from the CCR transformed TVC model (9) and from the same regression augmented with s additional superfluous regressors. Under the null hypothesis that the true model is a TVC cointegration model, the limit distribution of the test τ^* is a chi-square with s degrees of freedom. The basic idea underlying the test τ^* is to exploit the tendency of the unit root processes to be correlated with superfluous variables

[6] Denote such residuals $(\hat{u}_{kt})$ and define $\hat{w}_{kt} = (\hat{u}_{kt}, v_t')'$. Then the components of the autocovariance function can be estimated nonparametrically as

$$\hat{\Omega}_{nk} = \frac{1}{n}\sum_{|j|\leq h_n} K\left(\frac{j}{n}\right)\sum_t \hat{w}_{kt}\hat{w}_{k,t-j}' \tag{11}$$

$$\hat{\Delta}_{nk} = \frac{1}{n}\sum_{0\leq j\leq h_n} K\left(\frac{j}{n}\right)\sum_t \hat{w}_{kt}\hat{w}_{k,t-j}' \tag{12}$$

where K is the kernel function and h_n the window width. Commonly used kernels, such as Barlett, Parzen or the rectangular kernel, would lead to consistent estimators.

[7] See Park and Hahn (1999, Theorem 10).

with deterministic or stochastic trend. If indeed the model (2) is spurious, it is well known that including such superfluous variables will significantly improve the fit of the regression, and therefore reduce the sum of squared residuals, even when they are known to be irrelevant. On the other hand, if the model (2) is an authentic cointegrating regression, the inclusion of such variables will hardly affect the estimation results. The choice of the superfluous regressors plays an important role for the actual performance of the test. In this paper, we use time polynomials $t, t^2, t^3, \ldots, t^s$ as superfluous regressors, following the suggestion given in Park (1990).

To motivate the statistic we will use for testing the validity of the TVC cointegration model (2) against the FC cointegration model (1), we note that the FC cointegration model (1) becomes a FC spurious regression if indeed the true cointegrating relation contains time varying coefficients. Hence, we may test for the validity of the model (2) against the FC cointegration model (1) simply by testing whether the FC model is cointegrated. For this, we define the statistic τ_1^*

$$\tau_1^* = \frac{RSS_{FC} - RSS_{FC}^s}{\varpi_*^2}, \tag{14}$$

analogously as τ^* given in (13), where RSS_{FC} and RSS_{FC}^s denote the sums of squared residuals, respectively, from the fixed coefficient cointegrating model (1)[8] and from the same regression augmented with s additional superfluous regressors, and ϖ_*^2 is an estimate of the long-run variance computed from the TVC cointegrating model (9).[9] Park and Hahn (1999) showed that the statistic τ_1^* has the same limiting null distribution as τ^*, and also that τ_1^* is a consistent test which diverges under the TVC cointegrating model.

Before proceeding to estimate the demand model for the Mexican case, we discuss how we may specify and estimate the variable z_t in our TVC model (2) that captures the seasonal component of the electricity demand.

2.3 Modeling Seasonality

We observe strong seasonality in the electricity demand, especially in high frequency data such as monthly, which needs to be properly modelled for the consistent estimation of the demand equation. Engle et al. (1989) have indeed shown that the parameters in a cointegrating regression will generally be inconsistent if the seasonality is stochastic. The standard approach to overcome such inconsistency problem has been to filter the data either by changing its periodicity (for example, from monthly to annual data) or by taking differences of the variables at the seasonal frequencies. The obvious consequences from using such solutions are reduction

[8] To deal with models with general error specifications the statistics τ_1^* are based on CCR transformed models.

[9] They show that the use of the long-run variance from the FC model reduces the divergence rate of the statistic, and recommend to use the more efficient estimate ϖ_*^2 computed from the TVC cointegrating model (9)

in the sample size and/or elimination of some long-run variations in the level variables, which we want to avoid due to data limitations and the slow convergence rate of the series estimators for the parameters in our TVC cointegrating model. An alternative approach is to directly model the seasonality by choosing a variable that captures the seasonal component of the electricity demand. Traditional candidates for such variable are temperature related-measures such as the number of heating and cooling days per period (for example, in a month) and the average temperature. Certainly, this approach will neither reduce the sample size nor eliminate the elements of the long-run variations in the data. However, this approach may introduce the risk of incorrectly estimating the effect of temperature on the rate of electrical equipment usage, if based on a broad temperature measure such as those mentioned above. For example, the use of an air conditioner is determined by the high temperatures during the day time, not by the overall daily average temperature. In this paper we take the latter approach, but with a new and more flexible temperature measure.

We assume that the seasonality of the electricity demand is mainly due to the weather conditions, and construct a variable that captures such seasonality using intraday temperature data. Specifically, we model the seasonality of the electricity demand using a temperature response function that relates seasonal variations of the demand with the current temperature levels. As a guide to construct such a function, we consider three general patterns that characterize the influence of the temperature on electricity demand. First, we observe that extreme temperatures, either high or low, increase the demand for electricity. This means we would see a U-shape graph, if we plot the demand versus current temperature. Second, we also observe that the response of the electricity demand to change in temperature is larger when the temperature is high compared to when it is low. That is, the response of the demand is different depending on the current temperature level. This phenomenon would be reflected in an *asymmetric* U-shape graph when the demand is plotted against the temperature. Third, when comparing the responses by sectors, we observe that the residential demand shows the strongest responsiveness, while the industrial sector demand exhibits the weakest responsiveness to the same change in temperature. These general characteristics are what we attempt to capture with a temperature response function, which will be used later to construct the seasonal component z_t of the demand. We assume that the temperature response function, say g, takes the FFF functional form given by

$$g(\tau_p) = c_0 + c_1\tau_p + c_2\tau_p^2 + c_3\cos(2\pi\tau_p) + c_4\sin(2\pi\tau_p) + \cdots , \tag{15}$$

where $\tau_p \in [0, 1]$ is the normalized temperature at time p (ideally with hourly data) and $c_i,\ i = 0, 1, 2, 3, \ldots$, are the parameters to be estimated.

Although in principle the parameters of the above temperature response function g could be estimated by regressing a measure of the seasonal component of the electricity demand against the terms on the right hand side of equation (15), the data limitations we face generally shape the way in which those coefficients are estimated in practice. It is important to notice that the temperature response function (15) is defined in terms of current temperature in order to extract the information about extreme temperatures and duration. In general any intraday data such as temperature readings taken every hour or every three hours can be used to estimate the function g. Although temperature data are available at such high frequencies, it is often difficult to find data for the electricity demand at a frequency higher than monthly. Consequently, we need to come up with a new measure computed from the available temperature data at the frequency that matches with the available electricity demand data. As the required measure, we use the "expected" value of the temperature response function computed over the period of time determined by the frequency of the demand data. It is given more explicitly by

$$\begin{aligned} \int_{p\in t} g(\tau_p) f_t(\tau_p) d\tau_p &= c_0 + c_1 \int_{p\in t} \tau_p f_t(\tau_p) d\tau_p + c_2 \int_{p\in t} \tau_p^2 f_t(\tau_p) d\tau_p \\ &+ c_3 \int_{p\in t} \cos(2\pi\tau_p) f_t(\tau_p) d\tau_p \\ &+ c_4 \int_{p\in t} \sin(2\pi\tau_p) f_t(\tau_p) d\tau_p + \cdots, \end{aligned} \tag{16}$$

where t is the period in which the demand data is indexed (for example, a month) and f_t is the density function of the temperature data over the period t. Notice that the density functions f_t of the temperature data are indexed by t, indicating that we allow the temperature densities to differ across different time periods to capture the changing weather conditions from one period to another in the same year or across different years.

Computing the terms in equation (16) is relatively straightforward. Using intraday temperature data (around 720 observations per month if the data is hourly or 240 if the data are collected every three hours) we can estimate the densities f_t by a non-parametric technique, such as kernel estimation, at each t. Once we obtain the estimates $\tilde{f}_t$ for the densities, we can easily compute the Riemann sum approximations of the integrals in the right hand side of the expected temperature response function given in (16). Finally, the estimates for the parameters in (16) are obtained by regressing the seasonal component of the electricity demand (as will be discussed in the empirical section) against a constant term and the temperature variables represented by some of the integrals on the right hand side of the expected temperature response function. That is, we estimate the regression

$$
\begin{aligned}
d_t^s &= c_0 + c_1 \int_{p\in t} \tau_p f_t(\tau_p) d\tau_p + c_2 \int_{p\in t} \tau_p^2 f_t(\tau_p) d\tau_p \\
&\quad + \sum_{i=1}^{q} \left(c_{2i+1} \int_{p\in t} \cos(i2\pi\tau_p) f_t(\tau_p) d\tau_p \right) \\
&\quad + \sum_{i=1}^{q} \left(c_{2(i+1)} \int_{p\in t} \sin(i2\pi\tau_p) f_t(\tau_p) d\tau_p \right) + \varepsilon_t, \qquad (17)
\end{aligned}
$$

where d_t^s is the seasonal component of the electricity demand, q the number of trigonometric pairs, ε_t the error term, and the other variables and parameters are defined as in (16). The OLS estimators $(\tilde{c}_0, \tilde{c}_1, \tilde{c}_2, \ldots, \tilde{c}_{2(q+1)})'$ of the parameters $(c_0, c_1, c_2, \ldots, c_{2(q+1)})'$ can thus be readily obtained. With these estimators we can construct an estimate $\tilde{g}$ for the temperature response function g defined in (15) as

$$
\begin{aligned}
\tilde{g}(\tau_p) &= \tilde{c}_0 + \tilde{c}_1\tau_p + \tilde{c}_2\tau_p^2 + \tilde{c}_3\cos(2\pi\tau_p) + \tilde{c}_4\sin(2\pi\tau_p) + \cdots \\
&\quad + \tilde{c}_{2q+1}\cos(2\pi\tau_p) + \tilde{c}_{2(q+1)}\sin(2\pi\tau_p), \qquad (18)
\end{aligned}
$$

and in turn use this to define the seasonal variable z_t of our TVC model (2) as

$$
z_t = \int_{p\in t} \tilde{g}(\tau_p)\tilde{f}_t(\tau_p) d\tau_p. \qquad (19)
$$

One advantage of our seasonality modelling described above is that it is general and flexible enough to encompass other approaches seen in the literature. For example, the approaches that use the first moment of the temperature distribution as their weather-related variable can be easily formulated in our framework simply by imposing the parameter restrictions, $c_i = 0$ for $i \geq 2$, in the temperature response function (15).

3. Empirical Implementation of the Model: The Mexican Case

3.1 Description of the Data

We use the TVC cointegrating model (2) to estimate the demand for electricity in Mexico for the residential, commercial and industrial sectors. Taking into account the fact that the estimators of the parameters in the TVC models converge at a slower rate compared to those in the usual FC models, we work with a relatively large sample consisting of the monthly data from 1985:01 to 2000:05, with 185 monthly observations in total. The electricity data are obtained from the *Comision Federal de Electricidad* (CFE), and include monthly sales and prices.[10] To identify the demand[11] by sector we follow the CFE's classification that categorizes the customers by their energy consumption. For instance, private customers who demand low voltage are classified into the commercial sector and those who demand medium to high voltage are classified into the industrial sector. In the case of the residential sector, CFE has a specific classification for these types of customers. It is important to mention that the reported CFE's data in general do not correspond to the energy consumption of the reported month because of the lag between the month when the consumption was realized and the month when the transaction was actually registered. This is mainly a result of the routine schedule of payments followed by the government. Accordingly we adjust the demand data before the estimation of the model.

In our study, we use a weighted average of electricity prices, since there are differences in the prices across regions of the country and among the levels of energy consumption per customer. Also, to analyze the impact of the price of substitute goods, such as natural gas and diesel, we use relative prices in our regressions.[12] As a proxy for the disposable income, we use private consumption[13] for the residential and commercial sectors, and an indicator for the industrial production that includes mining, manufacturing and construction for the industrial sector.[14] Regarding the frequencies of the data, the indicators for the industrial production are reported on a monthly basis, while the private consumption data are reported on a quarterly basis. To work with monthly data for our estimation, we therefore transform the quarterly consumption data into monthly, using as a pattern the behavior of the monthly industrial production index. Finally, given that Mexico is a relatively large country, we divide the country into five regions and collect the temperature data from their representative cities[15] to obtain the input variables for the temperature response function. Although the frequencies at which the temperature data are collected vary across regions and over time, we were able to obtain the temperature data taken every three hours for all regions and the months covered in our sample period.

3.2 Estimation of the Temperature Response Function

For the estimation of the temperature response function defined in (15), we need a measure for the seasonal component of the electricity demand, and estimates for the terms in the

[10] The sales of the CFE represents around 80% of the total in the country. The remaining 20% of the sales is from Luz y Fuerza, also a stated owned company, whose data were not completely available to us.

[11] Because the demand and supply of electricity are always in balance, we use without distinction the terms sales and demand throughout the paper.

[12] All prices are obtained from the components of the Producer Price Index generated by Banco de México.

[13] Here we implicitly assume that consumers first decide the amount of their income that is saved and consumed, and later they decide how much to consume of each good and service.

[14] All the real variables are obtained from INEGI (Instituto Nacional de Estadística, Geografía e Informática).

[15] Source of the temperature data, Comisión Federal del Agua, México.

Table 1: Temperature Response Functions

	Residential Sector		Commercial Sector		Industrial Sector	
	Coefficients	t-values	Coefficients	t-values	Coefficients	t-values
c_0	-0.096	-1.129	-0.241	-3.405	-0.042	-0.980
c_1	-1.251	-3.272	-0.188	-0.590	-0.410	-2.150
c_2	2.418	6.356	1.088	3.428	0.822	4.326
$\bar{R}^2$	0.829		0.801		0.706	

expected temperature response function given in (16). To estimate the terms in (16), we first estimate the temperature densities by kernel estimation, and use them to approximate the integrals in (16) for each region. Then, using the regional electricity consumption as weight, we obtain their weighted averages over regions, and use them as the terms on the right hand side of the expected temperature response function (16) for the whole country. A normal kernel with optimal[16] fixed bandwidth is used for the estimation of the density functions. On the other hand, as the measure for the seasonal component of the demand, we use the detrended series of the sectorial electricity sales, with the trend estimated as the centered 12 month moving average of the original series. Given that all of the involved variables are stationary, by nature (temperature) or by construction (seasonal component of demand), standard econometric techniques are applied to estimate the parameters of the regression (17).

In light of the three general observed patterns on the ways temperature influences the electricity demand, we find from our Mexican data that the best specification for the temperature response function (15) turns out to be a simple quadratic function of temperature with no trigonometric pair, i.e., $g(\tau_p) = c_0 + c_1\tau_p + c_2\tau_p^2$, for all sectors with the standardization $\tau_p = (s_p + 5)/50$ for the actual temperature s_p at time p. The OLS estimators for the parameters in the temperature response function for all sectors are presented in Table 1 and the shapes of the functions are shown in Figure 1. To facilitate the interpretations, they are converted as functions of the actual temperature s. Moreover, for easy comparisons across three sectoral demands, the estimated temperature response functions are given in terms of the deviations from the mean. If we denote by $\tilde{g}$ as earlier the estimated temperature response function of the standardized temperature, then the function $\hat{g}$ presented in Figure 1 can be written as $\hat{g}(s_p) = (\tilde{g}((s_p + 5)/50) - \bar{g})/\bar{g}$, where $\bar{g}$ is the mean value of the estimated temperature response function $\tilde{g}$, i.e., $\bar{g} = \int_0^1 \tilde{g}(\tau_p)\, d\tau_p$.

The shapes of the temperature response functions are as expected. They show the asymmetric U-shape in the range of temperatures that we observe, with the scale of the estimated parameters reflecting the fluctuations of the seasonal component of the demand around its

[16] Optimal in the sense that it minimizes the Approximation of the Mean Integrated Squared Error (AMISE) for normal kernels.

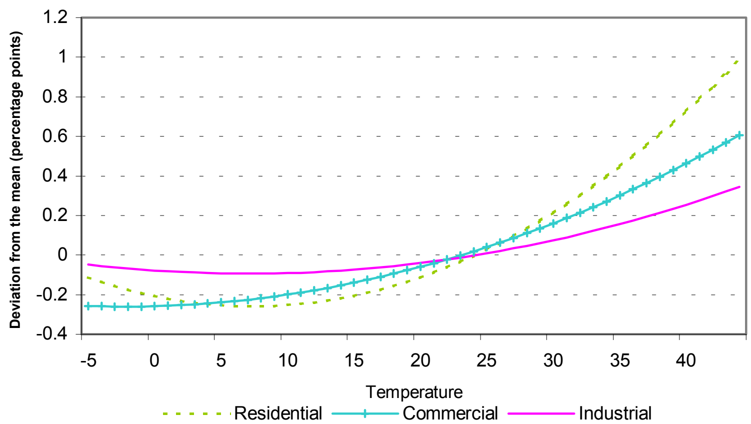

Figure 1: Temperature Response Functions by Sectors

mean. We illustrate in the following examples how one may interpret the results in Table 1. If we look at the residential sector at 25^0C, the estimated value of the response function is 0.0281 and at 26^0C, 0.0617. Hence, if there is an increase in the average temperature from 25^0C to 26^0C, the expected result is that the electricity demand will grow by around 3.27 percent.[17] On the other hand, if the temperature drops from 5^0C to 4^0C, the expected increase in the demand will be only 0.74 percent. For the industrial sector, the corresponding percentages for the same temperature references will be 1.17 and 0.18 percents, respectively, for an increase and a drop in the current temperature by one Celsius degree. These examples show the differences in the responses within the sector, which depend on the current temperature level, as well as the differences across the sectors, with the residential demand being the most responsive to the temperature variations. It is also worth noting from Table 1 that the squared temperature as well as the level temperature are important in explaining the seasonal patterns of the demand data. Indeed, the coefficient on the squared temperature is statistically significant, even the one on the level temperature is not, for the commercial sector temperature response function.

3.3 Estimation of the TVC Model

In order to properly specify the model for the estimation, we first analyze the nonstationary characteristics of the data. The presence of unit roots in the variables involved is tested using Augmented Dickey-Fuller (ADF) test with the shortrun dynamics determined by the Schwartz Information Criterion (BIC). According to the results reported in Table 2, there is evidence in

[17] Since the temperature response functions are given in terms of the deviations from their means, the percentage effect on the electricity demand of the temperature change from s_1 to s_2 may be computed as $(1 + \hat{g}(s_2))/(1 + \hat{g}(s_1)) - 1$.

favor of the presence of unit roots in the private consumption, the industrial production and all the electricity demand series. The results for the price series are, however, mixed and thus inconclusive, which is not surprising given the fact that the electricity prices in Mexico are controlled and heavily regulated by the CFE. Based on these results, we treat the income (y_t) and demand (d_t) series as known to be nonstationary variables, while we allow the price series (p_t) to be either stationary or nonstionary. Then we accordingly specify our TVC model as

$$d_t = \pi + \gamma_t y_t + \delta p_t + \phi z_t + u_t. \tag{20}$$

Notice that we allow the coefficient γ_t on the known to be nonstationary regressor y_t to vary over time to capture its evolving longrun relationship with the dependent variable d_t, which is also known to be nonstationary.[18] On the other hand, the coefficient δ on the potentially nonstationary regressor p_t is modelled as a fixed parameter. We note here that the CCR methodology, which we will use for the efficient estimation of our TVC model (20), is robust to the misspecification about the nonstationarity characteristics of the data, as shown in Kim and Park (1998). Thus, we may just regard the variable p_t, whose nonstationary characteristics is uncertain, as nonstationary and directly apply the CCR procedure introduced in Section 2.2.

We specify the time varying coefficient γ_t as a smooth function as in (5), and approximate it by a series of functions that include a constant, a linear time trend and k trigonometric pairs, as in (5). To determine the number k of the trigonometric pairs to be used in the series estimation of the time varying coefficient γ_t, we use BIC to pick a parsimonious model since it is known to favor simpler models by giving heavier penalties to the models with larger number of parameters. The CCR transformations are based on the differences of the detrended y_t and p_t, and the nonparametric estimators of the long-run variance Ω and the one-sided long-run variance Δ matrices defined in (11) and (12).[19] Table 3 reports the results obtained after removing the statistically insignificant variables from the models.

Before we analyze the results of our model (20), we first examine the validity of the model. To this end, we use the specification tests τ_1^* and τ^* introduced in Section 2.2, which are constructed here by using four time polynomial terms (t, t^2, t^3, t^4) as the additional superfluous regressors. Table 4 reports the results from estimating the fixed coefficient model needed to construct the test τ_1^*, while Table 5 presents the computed test statistics τ^* and τ_1^* for all sectors. The results of τ_1^* clearly show that the FC cointegrating model is rejected in favor

[18]Working with logs of the series, γ_t can be regarded as the "instantaneous elasticity" of d_t with respect to y_t. To see this, notice that taking the total differential and fixing p_t and z_t we have

$$\mathrm{d}d_t = \gamma_t \mathrm{d}y_t + y_t \mathrm{d}\gamma_t \quad \text{or} \quad \frac{\mathrm{d}d_t}{\mathrm{d}y_t} = \gamma_t + \frac{y_t}{\mathrm{d}y_t}\mathrm{d}\gamma_t$$

As $\Delta t \to 0$, $\mathrm{d}\gamma_t \to 0$ faster than $\mathrm{d}y_t \to 0$ (due to the smoothness assumption on γ_t) and therefore $\mathrm{d}d_t/\mathrm{d}y_t \to \gamma_t$.

[19]For the nonparametric estimation, we used the Parzen window with the lag truncation number selected by using the data-dependent selection rule suggested by Andrews (1991).

Table 2: Unit Root Tests (Augmented Dickey-Fuller Tests)

Variables	Demeaned Series	Lags	Detrended Series	Lags
d_t				
Residential	-1.183	12	-0.903	12
Commercial	0.250	13	-1.997	13
Industrial	0.550	12	-1.180	12
y_t				
Private Consumption	0.212	16	-1.914	16
Industrial Production	-0.221	2	-3.167	2
p_t				
Residential	-2.876	12	-3.172	12
Commercial	-4.105	1	-3.711	1
Industrial	-2.170	2	-3.515	1
5% Critical Values	-2.860		-3.41	

Table 3: CCR Estimation of TVC Regression Models

	Coefficients by Sector					
Variables	Residential		Commercial		Industrial	
Constant (π)	6.280	(4.04)	6.841	(4.70)	10.33	(7.92)
Price (δ)					-0.048	(-3.6)
z_t (ϕ)	0.995	(28.4)	0.995	(29.5)	0.870	(15.5)
Parameter Estimates of the TVC's: γ_t						
k	1		2		2	
$\beta_{k,1}$	0.367	(4.81)	0.289	(4.04)	0.226	(3.34)
$\beta_{k,2}$	0.039	(20.5)	0.026	(15.2)	0.041	(20.7)
$\beta_{k,3}$	-0.002	(-8.0)	-0.001	(-5.0)	0.0018	(9.21)
$\beta_{k,4}$	-0.0006	(-1.5)	0.0008	(2.17)	0.0016	(5.43)
$\beta_{k,5}$			0.0014	(5.25)	-0.0017	(-6.9)
$\beta_{k,6}$			0.0003	(1.23)	-0.0006	(-2.3)
Longrun Variances of the CCR Errors						
Ω^*_{11}	0.00278		0.00145		0.00079	
SC	-5.71		-5.90		-6.84	
$\bar{R}^2$	0.975		0.957		0.99	
DW	1.47		2.69		1.56	

Note: The numbers in parentheses are the t-values based on the CCR estimators.

Table 4: CCR Estimation of FC Regression Models

		Coefficients by Sector					
Variables		Residential		Commercial		Industrial	
Constant	(π)	-25.82	(-9.64)	-15.73	(-16.84)	-9.99	(-7.80)
Price	(δ)	-0.44	(-3.47)	-0.07	(-1.09)	-0.25	(-5.82)
Income	(γ)	1.95	(14.93)	1.40	(30.80)	1.29	(19.57)
z_t	(ϕ)	0.70	(5.09)	0.83	(11.63)	0.43	(1.59)
Longrun Variances of the CCR Errors							
Ω_{11}^*		0.0406		0.0068		0.0203	
SC		-4.212		-5.041		-5.028	
$\bar{R}^2$		0.875		0.875		0.928	
DW		1.097		1.867		0.923	

Note: The numbers in parentheses are the t-values based on the CCR estimators.

Table 5: Specification Tests

Sectors	τ_1^*	τ^*
Residential	663.97	6.74
Commercial	438.73	2.47
Industrial	1042.36	11.66
1% Critical Values	13.28	13.28

of the time varying coefficient (TVC) cointegrating model (20). The statistic τ^* also suggests that the TVC model is well cointegrated in all sectors at one percent significance level. The evidence is especially strong for the commercial and residential sectors. Notice that in the case of the fixed coefficient model, which specification was rejected, the elasticity of the electricity demand with respect to the income or production is higher than one, and the price elasticity is significant in the residential and industrial sector. Now that our TVC model (20) is tested to be an authentic cointegration model, we may now meaningfully interpret the coefficient estimates reported in Table 3 as the parameters of the longrun relationships among the variables.

We first note that contrary to the results of the fixed coefficient model, the parameter estimates for the prices are not statistically significant in the demand functions for the residential and commercial sectors. This is still the case even when we include as explanatory variables the indices for the relative electricity prices with respect to the prices of its close substitutes, such as natural gas for domestic use. One possible explanation for this finding is the price distortions from the government subsidies to the electricity prices. Indeed there have been large amount of governmental subsidies in most of the developing countries. In Mexico, for instance, a government report estimates the implicit subsidies to the electricity prices during the year

2000 is in the amount of 4.5 billions of dollars, which amounts to an overall 31 percent subsidy to Mexican electricity prices.[20] The subsidy to the residential customers was approximately 61 percent, while the subsidy to the commercial and industrial customers was only 7 percent with respect to the real cost of electricity. These figures justify very well the lack of explanatory power of the prices in the residential demand for electricity. When there is large amount of subsidy in electricity prices, prices would naturally become weak determinants of the demand for electricity. In such cases, the factors related to electricity availability would become more relevant.

In the commercial sector, on the other hand, much of the burden from the price increases can be treated as cost and eventually passed on to final customers, thereby generating a significant amount of relief for the commercial customers. This along with the lack of flexibility to use alternative energy sources in the commercial sector may explain why the prices are not significant determinants for the commercial demand for electricity. Also in the industrial sector, we find that the electricity demand does not respond to the electricity prices. However, it turns out that the industrial demand responds to the relative price with respect to the price of diesel, although it did not respond to other relative prices related to the price of natural gas. These findings suggest that generators run by diesel were the main back source of the electricity used by the industrial plants for the whole sample period from 1985 to 2000. It is also consistent with the fact that the use of the generators run by natural gas was promoted only in the last few years. According to our results in Table 3, the price elasticity of the industrial electricity demand is around -0.05, which is not nearly as big as those reported for other countries in earlier studies relying on the models with fixed coefficients,[21] and also those produced by the fixed coefficient model estimated from our Mexican data. See Table 4. In general it is observed that once the TVC is introduced, the estimates for all coefficients in the electricity demand equation become lower than those obtained from the usual FC models used in other studies (see, Westley (1992)).

The estimated values of the time varying coefficient γ_t on the income variable y_t are presented by sector at some representative months in Table 6, and Figure 2 plots these coefficients for the whole sample period. One consistent result comes out of these estimations is that for all sectors the TVC follows a predominantly increasing path during the entire fifteen years of our study. Since the values of γ_t are less than one but increasing, we may say that in all sectors the demand for electricity is becoming less inelastic with respect to the variable y_t (private consumption for the residential and commercial sectors, and industrial production for the industrial sector). This suggests either that the stock of electrical equipments and appliances was becoming less efficient in the consumption of energy,[22] or that the new ones are more intensive

[20] SHCP, Source: La Jornada, newspaper, Mexico, March 14, 2001.
[21] They are between -0.5 and -1. See, for instance, Westley(1992), pp 86.
[22] For example, consider the case when the electrical equipment and appliances are becoming old.

Table 6: Values of the TVC's (γ_t)

	Sectors								
	Residential			Commercial			Industrial		
Coefficients	Values	a*	b*	Values	a*	b*	Values	a*	b*
$\gamma_{1985:02}$	0.37			0.29			0.23		
$\gamma_{1990:01}$	0.38	4.2	4.2	0.30	2.8	2.8	0.24	6.6	6.6
$\gamma_{1995:01}$	0.39	3.7	8.0	0.31	2.8	5.7	0.25	3.7	10.6
$\gamma_{2000:01}$	0.40	2.2	10.4	0.31	2.9	8.7	0.27	6.3	17.5
$\gamma_{2000:05}$	0.40	0.2	10.6	0.32	0.3	9.0	0.27	0.3	17.9

a* Increments with respect to the previous values (%)
b* Accumulated increments (%)

in the use of electricity. These perspectives are related to the growth sources of the economy and the characteristics of the stock of the electrical equipments. In any case, the changes that have occurred in the values of the time varying income elasticities seem irreversible.

Across the sectors, however, we find remarkable differences in the behaviors of the estimated time varying coefficients γ_t (income elasticities), and also in the paths they have taken over the sample period. See Table 6. In the residential sector, the total increments in the values of the income elasticities over the entire sample period is 10%. The path of the elasticity also shows a clear tendency of slowing down. The increments are reduced from 4% in the first five years of the sample to only 2.2% in the last five years, from January 1995 to January 2000. On the other hand, the income elasticity of the commercial electricity demand increased on average 2.8% every five years, and by 8% for the entire fifteen years of the study. We observe much more noticeable variations in the estimates for the TVC (production elasticity) in the industrial sector. The total increments in values of the TVC here is approximately 18% over the entire sample period, which is far greater than those in the residential and commercial sectors. In May 2000, the last period of our sample, the value of the production elasticity reaches the level 0.267; however, it is still much smaller than the figures reported in other studies on the electricity demand for Latin America and the US. They report essentially unitary production elasticity, see Westley (1992, p 87) for example.

In the industrial sector, we find similar patterns of the estimated values of the production elasticity for two sub-periods: the first from 1985 to 1992, and the other from 1993 to 2000. In each sub-sample, the values of the production elasticity increases rapidly for the first 3-4 years, but the increments become smaller for the subsequent years and eventually are stabilized. In both sub-samples, the values of the production elasticity increased by around 6.5%. It is interesting to note that the beginning of the rapid increase in the production elasticity coincides with the periods of recession in the economy (1985 and 1993) as well as with two

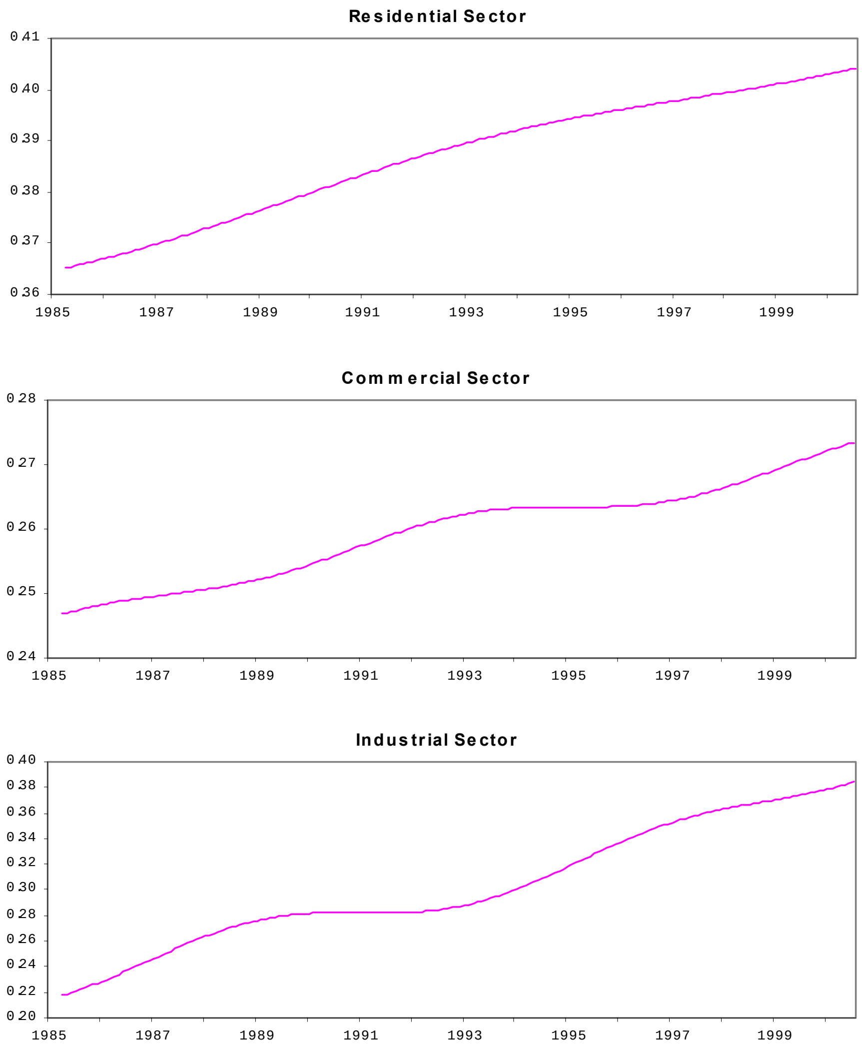

Figure 2: Time Varying Coefficients by Sector

events that significantly impacted the industrial structure: the inclusion of Mexico to the General Agreement on Tariff and Trade (GATT) in 1985 and the negotiation of the North America Free Trade Agreement (NAFTA) in 1993. The recessions may have been reflected in low production levels that did not allow an efficient use of the electrical equipments. If that was the case, then the electricity demand response is different depending on the state of the economy: the increment in electricity demand due to an increment in production would be higher when the economy is in recession than when the economy is in expansion. However, the increasing path of the production elasticity would be better explained in terms of the specific characteristics of the industry. In this regards the inclusion of Mexico to the international trade agreements may have triggered industrial expansion, which in turn triggered an intensive use of electricity.

Besides its ability to allow for flexible coefficients, the TVC model also provides a tool for forecasting the future path of the coefficients. The function that is used to model the time varying income elasticity takes the form given in (5), which involves only deterministic functions. Hence, we may readily obtain the time profile of the time varying coefficient, as reported in Table 3, and use them to forecast the path of the income or production elasticity for the forthcoming months.

3.4 ECM and Forecast Comparison

The ECM is formulated based on the TVC cointegrating model of the electricity demand specified in (20). From the results on the CCR estimation of the TVC model (20), we first obtain the following error correction terms

$$ec_t = d_t - \hat{\pi}^* - \hat{\gamma}_t^* y_t - \hat{\delta}^* p_t - \hat{\phi}^* z_t,$$

which is nothing but the fitted residuals, and where the values of $\hat{\pi}^*, \hat{\gamma}_t^*, \hat{\delta}^*$ and $\hat{\phi}^*$ are reported in Table 3. To focus our analysis on the dynamics of the electricity demand growth driven by those of the economic factors only, we define the following mean and seasonality adjusted demand series

$$\tilde{d}_t = d_t - \hat{\pi}^* - \hat{\phi}^* z_t,$$

which extracts the component of the electricity demand explained by income/production and prices (when they appear to be statistically significant).

Then we formulate an ECM for $\tilde{d}_t$ as

$$\Delta \tilde{d}_t = \sum_{k=1}^{q} b_{1k} ec_{t-k} + \sum_{k=1}^{p_1} b_{2k} \Delta \tilde{d}_{t-k} + \sum_{k=1}^{p_2} b_{3k} \Delta y_{t-k} + \sum_{k=1}^{p_3} b_{4k} \Delta p_{t-k} + \varepsilon_t. \tag{21}$$

Note that here we may have more than one lags of the error correction term ec_t as the right hand side variables. This is in contrast to the usual ECM based on a FC cointegration model,

Table 7: Error Correction Coeficients

	Sectors					
	Residential		Commercial		Industrial	
Coefficient b_{11}	-0.918	(-6.487)	-0.759	(-5.526)	-0.791	(-5.520)

Note: The numbers in the parentheses are the associated t-values

for which only one lagged error correction term is allowed due to the multi-collinearity problem between its lags and the lagged differences of the other included variables. In our ECM (21), the presence of the time varying coefficient (γ_t) in the lagged error correction terms (ec_{t-k}) eliminates such multi-collinearity problem. The coefficients (b_{1k}) of the lagged error correction terms (ec_{t-k}) characterize the adjustment path over q time periods toward the equilibrium value of the demand after an external shock. Thus, we are now able to learn more about the adjustment path, including how long it takes to reach the long-run equilibrium. For estimation, we set the lag orders p_1, p_2 and p_3 at 12 since the data are monthly, and focus our attention on the statistical significance of the coefficients (b_{1k}) on the lagged error correction terms, which are the new feature of the model. To determine the number of lags q of the error correction terms, we estimated the model (21) for several choices of q. We found that including more than one lag of the error correction term tends to make all the coefficients (b_{1k}) statistically not different from zero. However if we include only the first lag, then the coefficient b_{11} becomes significantly different from zero with the expected negative sign, which is consistent with a stable adjustment process toward the long run equilibrium. The result suggests that the adjustment of the electricity demand is concentrated in the period that follows immediately after an external shock. The value of the estimated error correction coefficient b_{11} and the associated t-value are reported in Table 7 for each sector.

Finally, we conduct an out-of-sample forecast exercise for each sector to evaluate the performance of our proposed TVC cointegration model (TVCCM) in (20) in relation to those of the FC cointegration model (FCCM) in (4), and the ECM derived from our TVC cointegration model (ECM) in (21). The data are divided into two subsets: the first set covering the observations from 1985:01 to 1999:05 is used to fit the model, and the second set, including those from the remaining twelve months (1999:06 - 2000:05), is reserved for the forecast evaluation. For each model and sector, the given model is estimated using the first subset of the data, and subsequently the forecasts for the last twelve months are computed using the estimated parameters, along with the actual data on the exogenous variables, y_t, p_t, and z_t, and the forecasted values of the time varying coefficients $\hat{\gamma}_t$. The forecasts from the three competing models, FCCM, TVCCM, and ECM, are presented in Table 8 along with the actual values of the observed electricity demand from the second subset of the data. We also compute the root

Table 8: Out-of-Sample Forecast Comparison between FCCM, TVCCM and ECM

	Residential Sector				Commercial Sector				Industrial Sector			
		Forecast				Forecast				Forecast		
Period	d_t	FCCM	TVCCM	ECM	d_t	FCCM	TVCCM	ECM	d_t	FCCM	TVCCM	ECM
1999:06	14.807	14.876	14.720	14.713	13.404	13.455	13.428	13.379	15.671	15.717	15.636	15.641
1999:07	14.837	14.785	14.681	14.744	13.502	13.389	13.398	13.468	15.649	15.720	15.629	15.616
1999:08	14.787	14.831	14.749	14.779	13.405	13.427	13.443	13.436	15.650	15.716	15.652	15.676
1999:09	14.679	14.758	14.667	14.633	13.421	13.370	13.390	13.424	15.621	15.701	15.629	15.616
1999:10	14.542	14.807	14.569	14.528	13.279	13.404	13.331	13.265	15.582	15.653	15.592	15.596
1999:11	14.589	14.689	14.494	14.507	13.296	13.350	13.270	13.284	15.591	15.645	15.572	15.575
1999:12	14.563	14.631	14.461	14.498	13.227	13.293	13.227	13.198	15.575	15.646	15.564	15.523
2000:01	14.513	14.512	14.471	14.477	13.314	13.219	13.237	13.289	15.656	15.656	15.578	15.572
2000:02	14.550	14.585	14.535	14.522	13.303	13.281	13.292	13.266	15.625	15.679	15.603	15.584
2000:03	14.650	14.825	14.659	14.639	13.416	13.466	13.396	13.417	15.699	15.816	15.657	15.641
2000:04	14.763	14.786	14.708	14.676	13.446	13.434	13.424	13.389	15.709	15.753	15.667	15.650
2000:05	14.845	15.038	14.805	14.769	13.551	13.576	13.500	13.510	15.733	15.852	15.709	15.703
RMSE		0.119	0.071	0.062		0.067	0.047	0.030		0.073	0.033	0.043

mean square error (RMSE) for each model and sector as a basis for the comparison.

Based on the RMSE criterion, the TVC cointegration model clearly outperformed the FC cointegration model in all three sectors. The forecasts produced by the TVCCM have much smaller RMSE's compared to those produced by the FCCM. Indeed, the use of the TVCCM leads to the reduction in the RMSE by 30 to 55%. The results from comparing the models TVCCM and ECM are, however, mixed. In the residential and commercial sectors, the ECM predicts better the path of the electricity demand than the cointegration model, and overall the demand forecasts produced by the ECM follow closely the observed levels. This is not surprising since our forecast horizon is relatively short and the ECM is designed to explain the short-run behavior better than the cointegration model which focuses on modelling long-run relationship.

In the industrial sector, however, the TVC cointegration model performs better than the ECM. The RMSE obtained from the TVC cointegrating model is smaller than the one from the ECM, as it is shown in Table 8. This result suggests that the adjustment of the industrial electricity consumption is more responsive to the long term changes than to the temporal shocks in the industry. Part of the reason for this pattern is the inflexibility of the industrial equipments to switch among different energy sources. Another reason may be the lack of available alternative energy sources, such as natural gas, which is not readily available in Mexico because of the limited infrastructure for its distribution. However, we expect that there will be more economic incentives to adjust the industrial electricity consumption more promptly and continuously, as the market becomes more competitive and the substitutes for electricity become more available. An external shock would then have not only the lasting longrun impact but also the short-run adjustment effect on the pattern of the industrial electricity demand.

4. Concluding Remarks

In this paper we analyze the electricity demand for the residential, commercial and industrial sectors in Mexico, using a new approach by Park and Hahn (1999) that allows for time varying coefficients (TVC) in the cointegrating relationships in electricity demand equation. The demand equation is specified in a double log functional form with income/production, electricity prices and a measure for seasonal fluctuations as explanatory variables. The income/production elasticity is in particular specified as a TVC, and with this flexibility we may model the evolving relationship between the electricity demand and the level of income/production, which was not possible in the previous models with fixed coefficients. To reflect the intrinsically slow changes in the factors affecting the electricity demand, such as technology, developmental stage of the economy and habit persistency, the TVC is modelled as a smooth function and approximated by a series of functions. As a measure for the seasonality in our monthly electricity demand data, we use a seasonal variable constructed from the temperature response function which is estimated nonparametrically from intraday temperature data.

Validity of the proposed TVC cointegration model specification is tested against the alternatives of the spurious regression and the fixed coefficient cointegration model, using the Wald type variable addition tests. The specification tests support the TVC cointegration model against both of the alternatives, favoring the notion of a changing relationship between electricity demand and its main determinants over time. The resulting TVC cointegration model is estimated by the cannonical cointegration regression method to deal with the endogeneity and serial dependence of the error resulted from the aforementioned approximation.

We find the following three evidences from the estimation of the TVC cointegration model for the electricity demand. First, the inclusion of the TVC in the model significantly reduces the levels of the estimated coefficients, which represent the elasticities, compared to those obtained from the usual fixed coefficient models. In particular, the income/production elasticity is substantially less than unity for all sectors, indicating that the demand is inelastic with respect to the changes in income/production in all sectors. This is quite surprising, since many earlier studies in the literature reported that the income/production elasticity of the electricity demand is near unity. Second, in the last fifteen years such elasticities have taken predominantly increasing paths in all sectors, and do not show any evidence of returning to their previous levels. Third, once we assume a TVC in the demand equation, price becomes statistically irrelevant as an explanatory variable for the long run behavior of the electricity demand, except for the industrial demand which weakly responded to the relative price of electricity with respect to the price of diesel. The lack of explanatory power of the prices in the electricity consumption of the residential and commercial sectors may be explained by the rigidities of the prices which have been determined and heavily subsidized by the government.

A direct consequence of having a TVC in the cointegration model is that the derived ECM can now include more than one lags of the error correction terms, thereby allowing us to learn more about the adjustment path toward the long-run equilibrium. Presence of a TVC in the error correction terms eliminates the multicollinearity problem observed in the usual ECM based on the fixed coefficient model. We finally conduct an out-of-sample forecast comparison between the TVC cointegrating model, the ECM derived from the TVC cointegration model and the usual fixed coefficient model. As expected, the TVC cointegrating model outperforms the fixed coefficient model in forecasting electricity demand for the twelve-month horizon in all sectors: sometimes reducing the root mean square error by more than 50%. Moreover, the ECM outperforms the TVC cointegrating model in the residential and commercial sectors, and this is also well expected since the forecast horizon is relatively short. However, the TVC cointegrating model performs better than the ECM in the industrial sector, which may be explained by the inflexibilities in the industrial equipments to swich among different energy sources and the limited infrastructure for the distribution of the alternative energy sources, hindering the industrial demand to adjust quickly.

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