Supply Side Structural Change*

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Abstract

Growing economies often exhibit constant growth rates, constant interest rates, and an increasing urban share of their population. We show that the equilibrium path triggered by a capital-biased technological revolution can account for these regularities. This type of technological change can generate an aggregate production function that displays linear segments. As the economy moves along those segments, the interest rate and the growth rate are constant, and labor is gradually reallocated from the old (rural) techniques to the new (urban) techniques. The model predicts that developed countries must experience a sudden slowdown in their growth rates once their structural change is completed. Productivity, as measured by the Solow residuals, also displays a growth slowdown. Cross-country evidence supports these predictions of the model.

JEL classifications: O14, O15, O18, O41, O47
Keyworkds: Growth, Structural Change, Urbanization, Cities, Choice of Techniques, Productivity Slowdown.

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1 Introduction

Models of growth are often judged for their ability to replicate to the so-called *Kaldor facts*. According to Kaldor (1963), growing economies usually display a constant growth rate of output, real interest rate, labor income share and capital-output ratio. Just as important as the *Kaldor facts* are the regularities about *structural change* documented by authors such as Clark (1940), Lewis (1954), Kuznets (1957,73), and Chenery (1965). They stress the fact that growing economies experience significant changes in the composition of output and the location of the population. For example, the percentage of urban population in currently developed countries has increased from around 11% in 1800, to 32% in 1900, to 65% in 1980 (Bairoch, 1988, Table 13.4).

Are current models of growth consistent with both set of regularities? A recent study by Kongsamut, Rebelo and Xie (1997) (henceforth, KRX) has shown that balanced growth models can account for the Kaldor facts but not for the massive sectoral reallocation. After all, they argue, balanced growth entails constant sectoral shares. They ask if a model that allows for non-homothetic preferences and sector-specific rates of technological progress can account for at least a constant interest rate and changes in the sectoral composition of labor and output. Their results are rather negative. Such path only exists under a particular initial distribution of capital across sectors, and a particularly unappealing configuration of preferences and technologies. They also conclude that sector-specific technological change is not enough to generate the structural change unless preferences are non-homothetic.

We offer an alternative explanation for the regularities that relies on technology rather preferences. We show that a capital-biased technological revolution naturally leads to an adjustment dynamic consistent with most of the *Kaldor facts* but also labor reallocation. More precisely, we introduce a new capital-intensive technique[1] in a world characterized by labor-intensive techniques. This 'industrial revolution' triggers a gradual process of technological adoption, capital accumulation, and labor reallocation from the *backward* technique to the *advanced* one. The *endogenous* aggregate production function becomes linear (takes an $AK + BL$ form) in spite of the fact that individual techniques are strictly concave in each input. As a result, the transitional path of this economy resembles the one of an AK model (Rebelo, 1990) characterized by a constant interest rate, and a

[1] We use the terms technique and technology as in Atkinson and Stigliz (1969). A technique is a *blue print* describing how inputs can be combined to produce a certain amount of output. Technology is the set of available techniques.

constant growth rate, but also labor reallocation.[2]

Our explanation for the regularities has at least two advantages compared with KRX. First, the initial distribution of capital is not crucial in our approach, and preferences and technologies can adopt standard functional forms. Second, structural change occurs regardless of whether the economy is open or closed. In our model, labor reallocation is just a necessary requirement in order to adopt new technologies. In contrast, the KRX explanation requires a closed economy. Only in that case changes in the composition of the demand, brought about by Engel's law, affect the supply side of the economy and the allocation of labor. This implication is important because the evidence indicates that most economies in the world have experienced structural changes independently of their degree of openness.

The model also offers a novel explanation for the well-known productivity slowdown. Growth in our artificial economy suddenly slows down once the structural change is completed. At that moment, capital accumulated in the advanced sectors cannot be matched with labor coming from the backward sectors, and as result decreasing marginal returns on capital set in. We show that growth accounting in our model would wrongly conclude that this slowdown results from a slowdown in productivity.

2 Related Literature

This paper shares the same spirit as Kongsamut, Rebelo and Xie (1997). They noted that balanced growth models are consistent with *Kaldor facts* but not with the observed reallocation of labor between activities. Our explanation for the regularities are, however, completely different. Their explanation relies on preferences, Engel's law, the demand-side of the economy, and balanced growth paths. Our explanation instead relies on technology, the supply-side of the economy, and transitional paths.

A second related paper is Zeira's (1998)[3]. He studies problems associated with the adoption of capital-biased innovations when countries differ in their productivity levels. We leave adoption problems aside, and focus on the adjustment path of an economy that has already decided to adopt the new technology. Such adjustment is instantaneous in Zeira's paper because his model economy is

[2] Glachant (2000) has also shown that an extension of this model can also produce a kind of Kuznetz curve: inequality jumps when the capital-intensive technique is introduced, monotonically rises during the structural change, and falls when the adoption of the new technique ends.

[3] The first version of this paper was written before Zeira's paper was published. Thus, some similar results were independently derived.

small and has full access to international capital markets. We assume an economy with no access to international capital markets, and as result the transition is slow and the interest rate is endogenously determined.

Other related papers are Hansen and Prescott (1998) and Goodfriend and McDermott (1995). They use models similar to ours but focus on very different issues to the ones in this paper. The production side of our economy follows the ideas of R. Jones (1974) about the endogenous choice of commodities by a small open economy. He studies the efficient choice of commodities by a country with fixed factor endowments. Output prices and technologies are determined by the world economy. We study a related question but allow for endogenous prices and factor accumulation[4]. Finally, Echevarria (1997) calibrates an economy similar to the one in KRX. Her model has similar limitations as KRX.

This paper is divided into seven sections. Section 3 reviews some important structural change regularities. Section 4 sets up the model and derives key results on the shape of the aggregate production function. Section 5 characterizes the equilibrium of the model and derives its implications. Section 6 elaborates some extension. Section 7 concludes.

3 Some Evidence of Structural Change

The rapid structural transformations of growing economies are among the most robust regularities of economic growth. These transformations include massive reallocation of labor from agriculture into manufacturing and services, and rapid urbanization. Kongsamut *et. al.* (1997) recently reviewed extensive evidence regarding the sectoral reallocation of labor and output for a wide set of countries. Their sample includes U.S. data for one hundred years, data for 22 countries for at least 50 years, and data for 123 countries for the period 1973-1989. They document that economic growth is accompanied by a significant decline of the agricultural sector, both as a share of the GDP, and as a share of labor employment.

Regarding the association between economic growth and urbanization, Bairoch (1988) presents abundant cross-country evidence from ancient times until 1980. He shows that the so-called 'Industrial Revolution' brought about a new era of economic growth characterized by a fundamental new role for cities, and an urban explosion. This process began in England, was then followed by some European countries and the U.S., and later by most other countries in the world.

[4] Another related study is Burmeister and Dobell (1970).

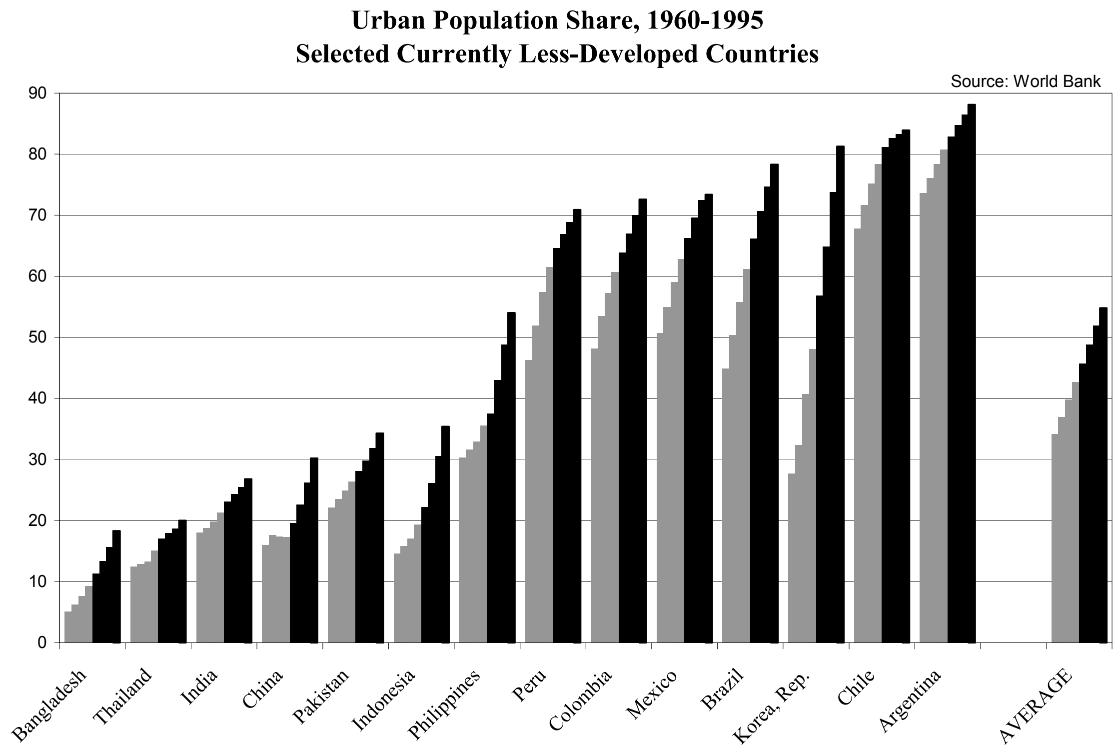

Figure 1: Share of Urban Population, Less-Developed Countries

Figures 1 and 2 display recent evidence about urbanization for 14 less-developed countries, and for 15 developed countries for the period 1960-1995. According to Figure 1, the urban share of population in less-developed countries has steadily increased and shows no signs of reaching a steady level. The urban share increased on average around 20 basic points during the period. In some cases, like Korea, it doubled in less than 30 years.

Figure 2 displays a different pattern of the urban share of population for developed countries. In those cases, the urban share steadily increased until the mid seventies at a similar pace as in the less-developed countries, but then the share almost stop growing. During the period 1960-75, the urban share rose 7.9 points in developed economies and 8.6 points in less developed countries. For the period 1975-95, the share rose around 12.1 points in less developed countries but only 2.2 points in currently developed countries.

In summary, we observe that growing economies experience an increase in their urban share of

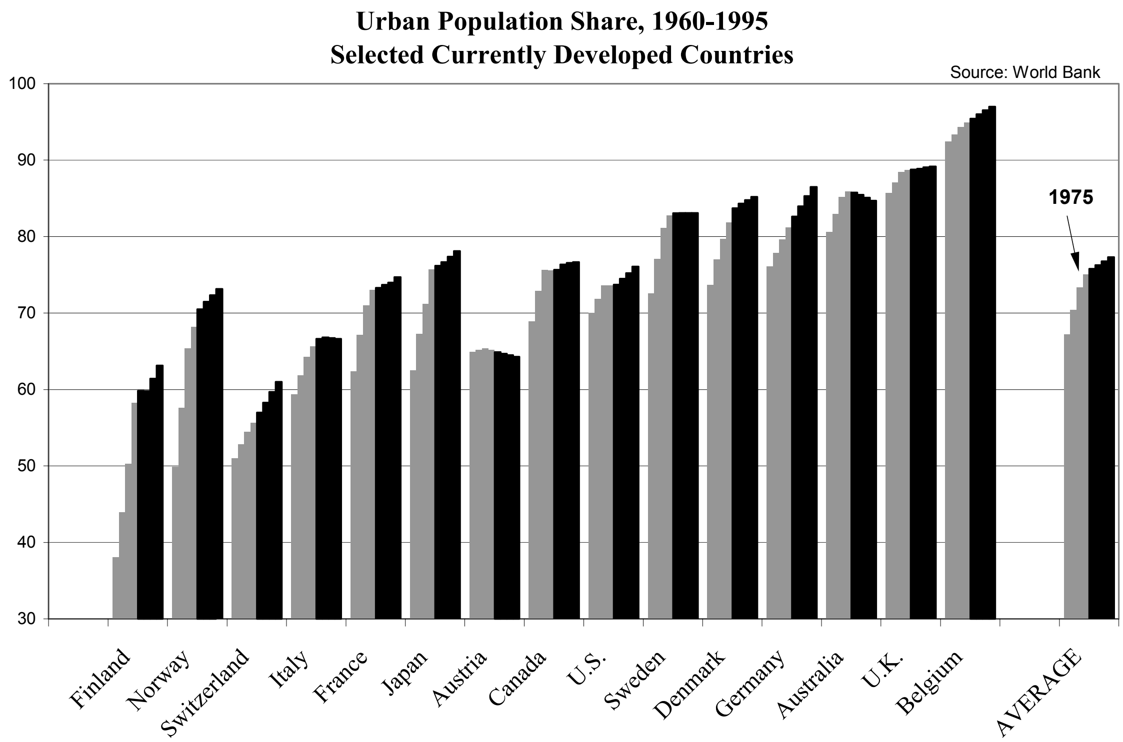

Figure 2: Share of Urban Population, Developed Countries

population, and that the urban share stops increasing once the economy reaches certain level of development.

4 The Model

Consider a closed economy that produces using a labor-intensive technique. At time zero the economy suddenly faces a drastic unexpected and biased technological change. A new capital-intensive technique is discovered. Although the new technique is more productive than the old one, it is not adopted immediately because capital is scarce or nonexistent at the moment of the invention.
We start by describing in detail the production possibilities, and then the preferences. For convenience, we present only the planner's problem but the results hold for a competitive equilibrium.

4.1 Technology

The economy produces only one good – which may be consumed or accumulated as productive capital–, two factors – capital and labor – and two techniques of production (after the discovery) – a *backward* one and an *advanced* one. We interpret those techniques as being rural and urban respectively. A precise model where this interpretation in meaningful is provided below as an extension of the model that includes land. Let K denote capital, L labor, and A a productivity parameter. Assumption 1 states that the technology satisfies standard requirements.

Assumption 1. The good in the economy can be produced using any combination of two C^2 linear homogenous techniques, $F^i(K_i, A_iL_i)$, $i = 1, 2$. Techniques display strictly positive first derivatives and are strictly concave in each argument. In particular, $\frac{\partial}{\partial x}\left(\frac{\partial F^i(x,y)}{\partial x}\right) < 0$ *and* $\frac{\partial}{\partial y}\left(\frac{\partial F^i(x,y)}{\partial y}\right) < 0$ *for $i = 1, 2$. In addition, $F^2(K_i, A_iL_i)$* satisfies the Inada conditions.

The following assumption states the existence of a capital-labor ratio threshold, $\widetilde{k}$, such that F^1 dominates F^2 for ratios below $\widetilde{k}$ while the opposite occurs for ratios above $\widetilde{k}$.

Assumption 2. (Unit isoquant crosses only once in R^{++}) $\exists$ a unique $\widetilde{k} > 0$ such that $F^1(\widetilde{k}, 1) = F^2(\widetilde{k}, 1)$. In addition, $F^1(K, L) > F^2(K, L)$ $\forall$ $K/L < \widetilde{k}$ and $F^1(K, L) < F^2(K, L)$ $\forall$ $K/L > \widetilde{k}$.

Assumption 2 guarantees two properties of the technology. First, no technique completely dominates the other. Second, reswitching is avoided, i.e. the case in which a technique that has been abandoned is re-adopted as the capital-ratio increases. In the light of the second assumption we call F^1 the backward or labor-intensive technique, and F^2 the advanced or capital-intensive technique. The single crossing property is a strong requirement. It is not satisfied, for example, by arbitrary combinations of Cobb-Douglas or CES production functions. This property, however, is not crucial for our results. If isoquants cross more than once, reswitching can be avoided if isoquants cross only once for capital-labor ratios below the steady state level.

Finally, we allow for labor augmenting technological progress and population growth.

Assumption 3. Technology and population evolve exogenously according to the rules

$$A_{it} = A_i e^{xt}, L_t = e^{nt}. \tag{1}$$

Figure 3 displays a pair of unit isoquants, one for each technique, satisfying Assumptions 1 and 2. The isoquants cross exactly once so that the backward technique is more productive – requires less inputs per unit of output – for low capital-labor ratios. This relation reverses for high capital-labor ratios. In the case in which techniques cannot be combined, the aggregate isoquant for the economy is just the envelope of the individual isoquants. In that case, it would be efficient to use the backward technique if the aggregate capital-labor ratio is below $\widetilde{k}$, and switch to the advanced technique if the aggregate capital-labor surpasses that level.

An alternative and relevant case for our purpose arises if techniques can be freely combined. In this case, factors are efficiently allocated among techniques, and the aggregate production function is defined by[5]

$$F_t(K_t, L_t) \equiv \max_{\substack{0 \leq K_1 \leq K \\ 0 \leq L_1 \leq L}} F^1(K_{1t}, A_{1t}L_{1t}) + F^2\left(K_t - K_{1t}, A_{2t}\left(L_t - L_{1t}\right)\right). \tag{*}$$

Let $k_{it} \equiv \frac{K_{it}}{e^{(x+n)t}}$ be the efficient capital-labor ratio allocated to technique i, measured in efficiency units of labor, and $k_t \equiv \frac{K_t}{e^{(x+n)t}}$ be the aggregate capital-labor ratio. Marginal products must be equal in any interior solution, i.e.,

[5] The maximum is well defined by Weierstrass Theorem.

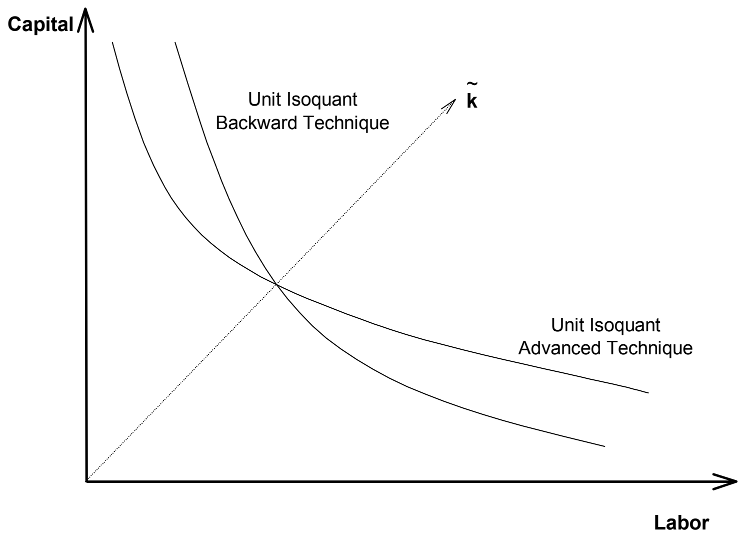

Figure 3: Techniques of Production

$$F_L^1(k_{1t}, A_1) = F_L^2(k_{2t}, A_2) \text{ and } F_k^1(k_{1t}, A_1) = F_k^2(k_{2t}, A_2). \tag{2}$$

The solution also must satisfy the aggregate constraint $K_t = K_{1t} + K_{2t}$, or

$$k_t = k_{1t} l_{1t} + k_{2t}(1 - l_{1t}), \tag{3}$$

where $0 \leq l_{1t} \equiv L_{1t}/L_t \leq 1$. Define $f^i(k_t) \equiv F^i(K_t, A_t L_t)/e^{(x+n)t} = F^i(k_t, A_i)$ for $i = 1, 2$. Using the fact that F^i is linear homogenous, and the equilibrium relations $K_1 + K_2 = K$, and $L_1 + L_2 = L$, we can recast (*) as:

$$f(k_t) \equiv \frac{F_t(K_t, L_t)}{e^{(x+n)t}} = \max_{\substack{k_1, l_1 \\ 0 \leq k_1 l_1 \leq k_t}} f^1(k_{1t}) l_{1t} + f^2\left(\frac{k_t - k_1 l_1}{1 - l_1}\right)(1 - l_{1t}) \tag{**}$$

The following proposition collects the main properties of the aggregate technology.

Proposition 1 *Let assumptions 1, 2 and 3 hold in problem (**). Then, there exist* $0 < \underline{k} < \overline{k}$ *such that (i) for* $k_t \leq \underline{k}$*, only technique 1 is used; (ii) for* $k_t \geq \overline{k}$*, only technique 2 is used; (ii) for*

$\overline{k} > k_t > \underline{k}$, *both techniques are used in such way that the* $k_1 = \underline{k}$ *and* $k_2 = \overline{k}$, *the fraction of total labor allocated to the second technique is*

$$l_{2t} = \left\{ \begin{array}{cc} 0 & \text{for } k_t < \underline{k} \\ = \frac{1}{\overline{k}-\underline{k}}\left(k_t - \underline{k}\right) & \text{for } \overline{k} \geq k_t > \underline{k} \\ 1 & \text{for } k_t > \overline{k} \end{array} \right\}, \quad (4)$$

and the aggregate production function satisfies

$$f(k_t) = \left\{ \begin{array}{cc} f^1(k_t) & \text{for } k_t < \underline{k} \\ \theta_1 + \theta_2 \cdot k_t & \text{for } \overline{k} \geq k_t > \underline{k} \\ f^2(k_t) & \text{for } k_t > \overline{k} \end{array} \right\}, \quad (5)$$

where $\theta_1 = A_1 F_l^1(\underline{k}, A_1) = A_2 F_l^2(\overline{k}, A_2)$ *and* $\theta_2 = F_k^2(\overline{k}, A_2)$.

Proof. See Appendix 1.1. ■

Corollary 2 *The aggregate production, F, is a differentiable linear homogenous function in* (K, L).

We now provide some intuition for these results. Note that (2) is a time homogenous system of two equations in two unknowns that can be solved for k_1 and k_2, *independently* of (3). Indeed, assumptions 1 and 2 assure that this system has a unique solution, and that $k_1 < k_2$. Let $\underline{k} \equiv k_1$ and $\overline{k} \equiv k_2$ be the solutions to (2). This pair is not necessarily the solution to (**) because (3) needs also to be satisfied. A brief inspection of the last equation, however, reveals that $(\underline{k}, \overline{k})$ is in fact the solution to (**) as long as $\underline{k} \leq k_t \leq \overline{k}$. It also follows that the marginal products of capital and labor remain constant as k_t moves along this interval.

Figure 4 presents a graphical description of these results. Since both techniques are linear homogenous, the aggregate unit isoquant is just the envelope of the convex combinations of the unit isoquants. As a result, the aggregate isoquant displays a linear segment in the region where techniques are combined. This region is known as the *cone of diversification (*Jones*, 1974),* a cone delimited by $\underline{k}$ and $\bar{k}$. Figure 4 also illustrates the efficient allocation of factors across techniques. Factors are allocated so that the capital-labor ratios in the backward and advanced techniques are $\underline{k}$ and $\bar{k}$ respectively.

It is apparent from Figure 4 that the aggregate production function is linear in the cone of diversification. It has the form $AK + BL$ in the cone, a result that resembles the AK model of Rebelo (1990). As in the AK model, our economy can also sustain unlimited endogenous growth if $\bar{k} \rightarrow \infty$. In that case, the marginal product of capital eventually becomes constant. In the case of $\overline{k}$ finite, the

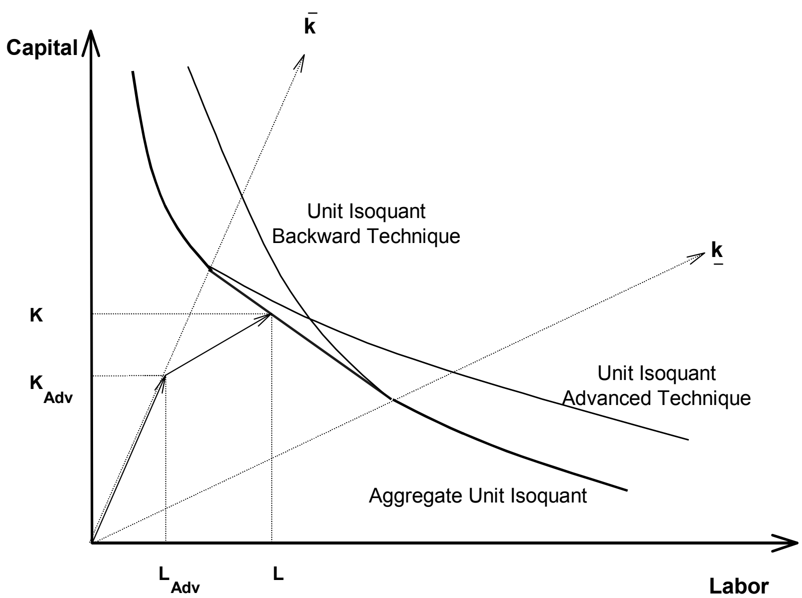

Figure 4: Aggregate Isoquant

economy eventually faces decreasing marginal returns, and endogenous growth is typically bounded[6]. The important observation is that growth along the cone of diversification inherits similar properties as the AK model, properties that we exploit below.

Consider the evolution of this economy as k_t increases. As long as the ratio is below $\underline{k}$, only the backward technique is employed and the marginal product of capital decreases. For $k_t \in [\underline{k}, \overline{k}]$, both techniques are employed. As k_t increases, a larger fraction of factors are allocated into the advanced technique, the marginal products of labor and capital are constant, and the elasticity of substitution is infinity. Finally, once the aggregate capital-output ratio exceeds $\bar{k}$, only the advanced technique is operated, and a decreasing marginal product of capital sets in again.

A useful example arises when the backward technique is linear in labor,

$$F^1(K_t, A_{1t}L_t) = A_{1t}L_t. \tag{6}$$

In this case $\underline{k} = 0$. Although this formulation does not satisfy Assumption 1, Proposition 1 still applies.

Lemma 3 *Let F^1 satisfy (6). Then $\underline{k} = 0$ and $\bar{k}$ is defined implicitly by the equation $F_l^2(\overline{k}, A_2) = \frac{A_1}{A_2}$. In addition $l_{2t} = \frac{k_t}{\bar{k}}$ for $\overline{k} \geq k_t$, $l_{2t} = 1$ for $k_t > \overline{k}$ and the aggregate production function satisfies:*

$$f(k_t) = \left\{ \begin{array}{cc} A_1 & \text{for } k_t = 0 \\ A_1 + f_k^2(\overline{k}) \cdot k_t & \text{for } \overline{k} \geq k_t > \underline{k} \\ f^2(k_t) & \text{for } k_t > \overline{k} \end{array} \right\}. \tag{7}$$

Proof. The marginal product of labor in the backward technique is A_{1t} and $A_{2t}F_2(K_t, A_{2t}L_{2t})$ in the advanced one. For $K_t > 0$, $\lim_{L \to 0} F_2(K, A_2L_2) = \infty$, then it is efficient to use the advanced technique whenever $K_t > 0$. Therefore $\underline{k} = 0$. We can recast the efficient condition to allocate labor in terms of the aggregate capital labor ratio and l_{2t}.

$$A_1 \leq A_2 \cdot F_2(k_t, A_2 l_{2t}^*) \qquad \text{with equality if } l_{2t} < 1. \tag{8}$$

$\overline{k}$ is the aggregate capital-labor ratio that makes $l_{2t}^* = 1$. Other results follow from Proposition 1. ■

4.2 Preferences

The planner seeks to maximize the utility of a representative, infinite-lived household

[6] Growth can still be unbounded with strictly decreasing returns on capital if they remain above a strictly positive lower bound.

$$\int_0^\infty e^{(n-\rho+(1-\theta)x)t} \frac{c_t^{1-\theta}}{1-\theta} dt, \tag{9}$$

where $c_t \equiv \frac{C_t}{L_t e^{xt}}$, C is the household consumption, L is labor (and population), n is the growth rate of labor, $\rho(> n)$ is the rate of time preference, and $\theta(> 0)$ is the (negative of the) elasticity of marginal utility. The planner faces the aggregate resource constraint

$$\frac{dk_t}{dt} \equiv \dot{k}_t = f(k_t) - c_t - (\delta + x + n)k_t. \tag{10}$$

We have followed the standard procedure of writing the utility function and the resource constraint in efficiency units of labor[7], i.e., dividing all level variables by $e^{(n+x)t}$.

5 Efficient Allocation

An optimal allocation for this problem must satisfy the following Euler equation,

$$\frac{\dot{c}_t}{c_t} = \frac{1}{\theta}\left(r_t - \rho - \theta x + \mu_t\right), \tag{11}$$

where r_t is the (net) marginal product of capital or interest rate in a decentralized equilibrium, μ_t is a lagrange multiplier equal to zero if $k_t > 0$ and positive if $k_t = 0$. The lagrange multiplier is required if capital is not essential for production, a case that occurs, for example, when F^1 is linear in labor.

The interest rate can be derived from (5) as given by

$$r_t = r(k_t) = \left\{ \begin{array}{cc} f_k^1(k_t) - \delta & \text{for } k_t < \underline{k} \\ \theta_2 - \delta & \text{for } \overline{k} \geq k_t > \underline{k} \\ f_k^2(k_t) - \delta & \text{for } k_t > \overline{k} \end{array} \right\}. \tag{12}$$

where $f_k(k)$ is the first derivative. It is remarkable that the interest rate is constant on the interval $\overline{k} \geq k_t > \underline{k}$. It is well known (see, for example, King and Rebelo (1993)) that the extreme response of the interest rate in the standard neoclassical model produces an unappealing transitional dynamics. In particular, the interest rate can be extremely high if the economy is far it is steady state. In

[7] Details can be found, for example, in Barro and Sala-I-Marti (1995).

contrast, in our model the marginal product of capital can be constant and remain in a sensible interval even for low levels of capital. For example, if $\underline{k} = 0$, the interest rate has an upper bound given by $r(0) = r(\overline{k}) = \theta_2 - \delta$.

5.1 Steady State Characterization

The steady state level of capital, k^*, is determined by (11) and (12). $k^* = 0$ if $f_k(0) < \delta + \rho + \theta x$, and otherwise it is implicitly determined by the equation

$$f_k(k^*) = (\delta + \rho + \theta x) . \tag{13}$$

If the condition $f_k(0) \geq \delta + \rho + \theta x$ holds, then a solution to (13) exist because F^2 satisfies the Inada conditions (Assumption 1). There are three cases to consider depending on which inequality holds true: $f_k(\overline{k}) \gtreqless f_k(k^*)$.

- *Case 1.* $f_k(\bar{k}) > \delta + \rho + \theta x = f_k(k^*)$. In this case $k^* > \overline{k}$ and $f_k(k^*) = f_k^2(k^*)$. It represents a case in which the economy only operates the advanced technique in steady state.

- *Case 2.* $f_k(\overline{k}) = \delta + \rho + \theta x = f_k(k^*)$. In this case there is a continuum of unstable steady states characterized by $\underline{k} \leq k^* \leq \overline{k}$. The scale at which both techniques are operated is undetermined. If the economy starts below $\underline{k}$ then it converges monotonically to $\underline{k}$, and if the economy starts above $\bar{k}$ then it converges monotonically to $\bar{k}$. If the initial level of capital lies between $\underline{k}$ and $\bar{k}$, the economy remains there forever. This multiplicity of the steady states could explain why economies with apparently identical preferences and technologies may perform very differently. In this case, initial conditions completely determine the steady state consumption, per-capita output, and capital levels.

- *Case 3.* $f_k(\overline{k}) < \delta + \rho + \theta x = f_k(k^*)$. This case represents an economy that only operates the backward technique in steady state In this case $k^* \leq \underline{k}$ and $f_k(k^*) = f_k^1(k^*)$.

It is important to stress that an economy that satisfies cases 1 or 3 possess a unique steady state. The particular point to which the economy converges depends on both technologies and preferences. For example, consider the simple case $F^1 \equiv A_1 L_{1t}$ and $F^2(K, A_2 L_2) \equiv K^\alpha (A_2 L_2)^{1-\alpha}$. The economy stays with the backward technique as long as[8]

$$\frac{A_1^{1-\alpha}}{A_2} > (1-\alpha)^{(1-\alpha)} \left(\frac{\alpha}{\rho + \theta x + \delta} \right)^{\alpha}. \tag{14}$$

On one hand, there are technological reasons. An economy that is particularly productive with the backward technique (large A_1), or particularly inefficient with the advanced technique (low A_2), a high rate of depreciation, or a high rate of labor augmenting technological progress can deter the adoption of the advanced technique. Preferences also play a crucial role. An economy with a high discount rate or high coefficient of risk aversion may prefer not to accumulate large amounts of capital.

How about taxes? Consider a tax on capital and labor income earned in the advanced sector. This seems an interesting case because low income countries usually tax primarily incomes earned at cities or formal sectors, and subsidize agricultural activities. Such income taxes has the same effect as a lower A_2 and higher A_1 and, therefore, can deter the adoption of advanced techniques.

The model also has a significant amplification effect in the sense that small changes in the parameters of the model can induce jumps in the steady state capital-output ratio. Such changes can affect the adoption decision, and drive the economy from one side of the cone to the other side. Consequently, countries with slightly different preferences, productivities, and/or taxes can display large differences in per-capita income[9]. In this case, government policies could be particularly powerful to push a country out of a low income level. These results contrast with the standard neoclassical predictions. In the standard model, steady state variables are continuous functions of the parameters, and government policies can only induce marginal changes in per-capita income.

How strong is the amplification effect? The additional growth in the model comes from capital deepening along the cone of diversification. Growth along the cone is given by

$$g^C \equiv \frac{f^2(\overline{k})}{f^1(\underline{k})} = \frac{F^2(\overline{k}, A_2)}{F^1(\underline{k}, A_1)}.$$

The following lemma establishes an important relationship between g^C and the capital share of income.

[8] We already show the advanced technique is eventually abandon if $k^* < \overline{k}$. We can rewrite this inequality using the definitions of k^* (from 13) and $\bar{k}$ (from 8), which provides the equation in the text.

[9] Zeira (1998) studies how this model amplifies differences in productivities. See footnote (3).

Lemma 4 *Let $\alpha(k)$ be the capital share of income when the aggregate capital-labor ratio is k. Then,*

$$g^C = \frac{1-\alpha(\underline{k})}{1-\alpha(\overline{k})}.$$

Proof. Using Euler rule for linear homogenous functions, g^C can be written as

$$g^C = \frac{F_L^2(\overline{k}, A_2)A_2 + F_k^2(\overline{k}, A_2)\overline{k}}{F_L^1(\underline{k}, A_1)A_1 + F_k^1(\underline{k}, A_1)\underline{k}}.$$

Along the cone, the marginal product of labor is constant so that $F_L^2(\overline{k}, A_2)A_2 = F_L^1(\underline{k}, A_1)A_1$. Therefore,

$$\begin{aligned} g^C &= 1-\alpha(\underline{k}) + \frac{F_k^2(\overline{k}, A_2)\overline{k}}{F_L^1(\underline{k}, A_1)A_1 + F_k^1(\underline{k}, A_1)\underline{k}} \\ &= 1-\alpha(\underline{k}) + \frac{F_k^2(\overline{k}, A_2)\overline{k}}{F_L^2(\overline{k}, A_2)A_2 + F_k^2(\overline{k}, A_2)\overline{k}} g^C \\ &= 1-\alpha(\underline{k}) + \alpha(\overline{k})g^C. \end{aligned}$$

Solving for g^C, the required result follows. ■

Thus, growth along the cone is fully reflected in the capital share of income, a natural result since the advanced technique is more capital-intensive. The larger the difference in capital intensities, as measured by α, the larger the multiplier. In fact, the multiplier can be very large if $\alpha(\overline{k})$ is close to 1, i.e., as the advanced technique becomes an AK technique. The popularity of AK models in the literature suggest that $\alpha(\overline{k})$ close to 1 is an important case to consider. The amplification effect in that case may be considerable.

It is important to stress at this point that the increasing capital share in our model is not inconsistent with the Kaldor observation that the labor share is constant. Capital may include physical and human capital, and as a result, part of the capital payments can in fact be accrued by the labor factor.

5.2 Transitional Dynamics: Kaldor Facts and Structural Change Regularities

The transitional path is determined by the steady state of the economy. We focus on the first case of the previous section in which the economy eventually adopts the advanced technique. Cases 2 and 3 can be analyzed in a similar way.

Assumption 4: $f_k(\overline{k}) > \delta + \rho + \theta x$.

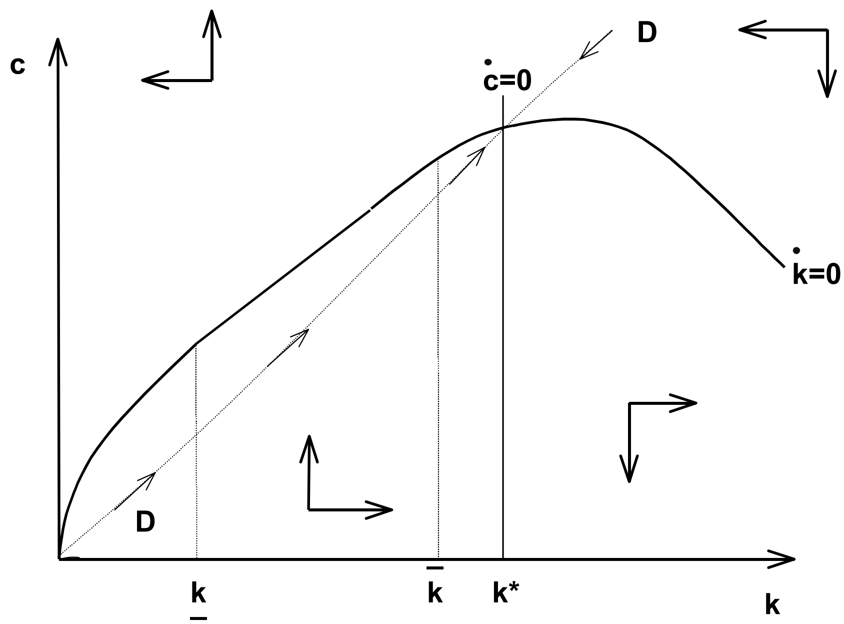

Figure 5: Phase Diagram

Using equations (11) and (12), we can rewrite the growth rate of consumption as

$$\frac{\dot{c}_t}{c_t} = \left\{ \begin{array}{ll} \frac{1}{\theta}\left(f_k^1(k_t) - f_k(k^*)\right) & \text{for } k_t < \overline{k} \\ \frac{1}{\theta}\left(f_k(\overline{k}) - f_k(k^*)\right) & \text{for } \underline{k} \leq k_t \leq \overline{k} \\ \frac{1}{\theta}\left(f_k^2(k_t) - f_k(k^*)\right) & \text{for } k_t \geq \overline{k} \end{array} \right\}. \tag{15}$$

Since $k^* > \bar{k}$, consumption grows as long as $k_t < k^*$, and it does at a constant rate as long as k remains in the interval $(\underline{k}, \overline{k})$. Above the cone of diversification, the economy behaves exactly as in the Ramsey model. In particular, consumption grows at a decreasing rate. Figure 5 illustrates a phase diagram for this case, constructed using equations (10) and (15), and the usual transversality condition. A look at the phase diagram reveals the following important lemma.

Lemma 5 *The optimal allocations of capital and consumption converges monotonically to their steady state levels. In particular, if $k_0 < k^*$, the model exhibits increasing sequences of capital, output, and consumption over time.*

Figure 6 displays the equilibrium path of three key variables in the model: the interest rate, described by equation (12), the growth rate of the economy, given by equation (15), and the share of labor in the advanced sector, described by equation (4). As we indicated, we interpret this share as the urban share of population. The following Proposition summarizes the "growth" properties of the model.

Proposition 6 *(Kaldor and structural change facts). Let technology and preferences satisfy Assumptions 1 to 4, and let* $k_0 < \bar{k}$*. Then, the equilibrium path of the economy goes through a transient period in which the growth rate of consumption growth and the interest rate are constant, the capital-output ratio and the capital-labor ratio increase, and the urban share of population gradually increases. Finally, the labor share of income may be constant.*

Proof. k_t increases monotonically toward k^* by Lemma 5 and the fact that $k_0 < \overline{k}$. As long as $k_t \in [\underline{k}, \overline{k}]$ the growth rate of consumption and the interest rate remain unchanged, and the urban share of population increases according to equations (15), (12), and (4).Since the production function $f(k^*)$ is concave, the average product, $f(k)/k$, is decreasing. As a result, the capital-output ratio increases along the transition. In addition, simple inspection of (5) reveals that $f(k)/k$ is strictly decreasing for $k \in [\underline{k}, \overline{k}]$. For $k_t > \bar{k}$ the ratio is also strictly decreasing since $f(k)$ is strictly concave. The labor share is constant if the larger capital intensity of the advanced technique reflects human capital. ■

Proposition 6 states that the transition path along the cone of diversification is consistent with most of the Kaldor facts, and with labor reallocation. The only major discrepancy concerns the capital-output ratio. According to Kaldor, growing economies exhibit a constant ratio but our model unequivocally predicts an increasing ratio. Our contend on this respect is that the evidence strongly indicates that the capital-output ratio has significantly increased in growing economies. For example, Zeira (1998, Section VIII) presented important evidence supporting this claim. Additional evidence can be found. Figure 7, for example, displays the non-residential capital-output ratios for the U.S. and for the set of currently developed countries considered by Maddison (1991, Table 3.10)) for the period 1890-1987. The sample includes France, Germany, Japan, Netherlands and the U.K. According to the evidence, the ratios can hardly be regarded as constant for any country. For example, the ratio increased by a factor of 1.88 for the set of "other countries" during 100 years.

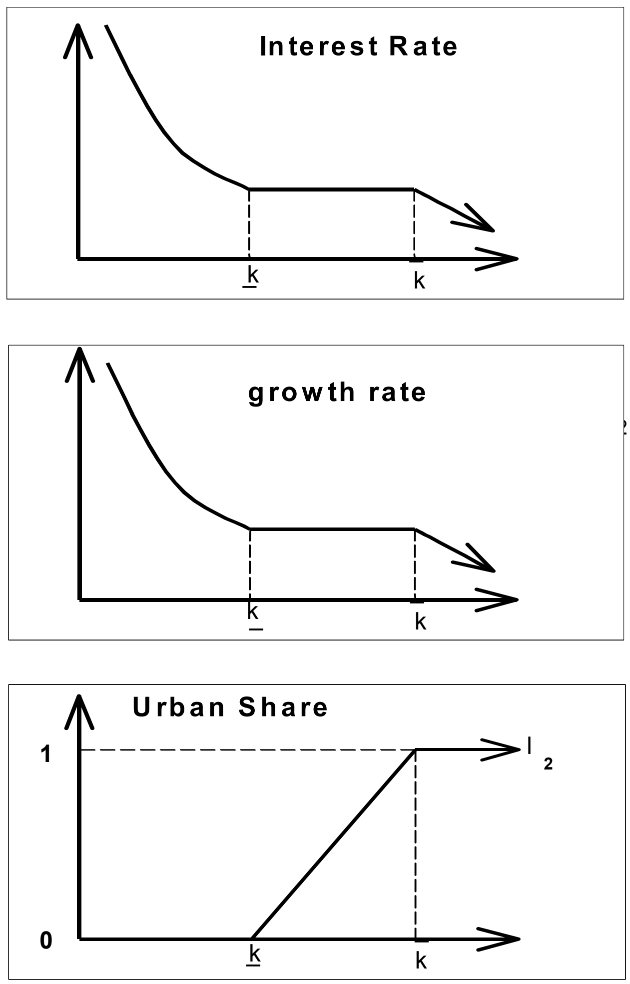

Figure 6: Key Variables

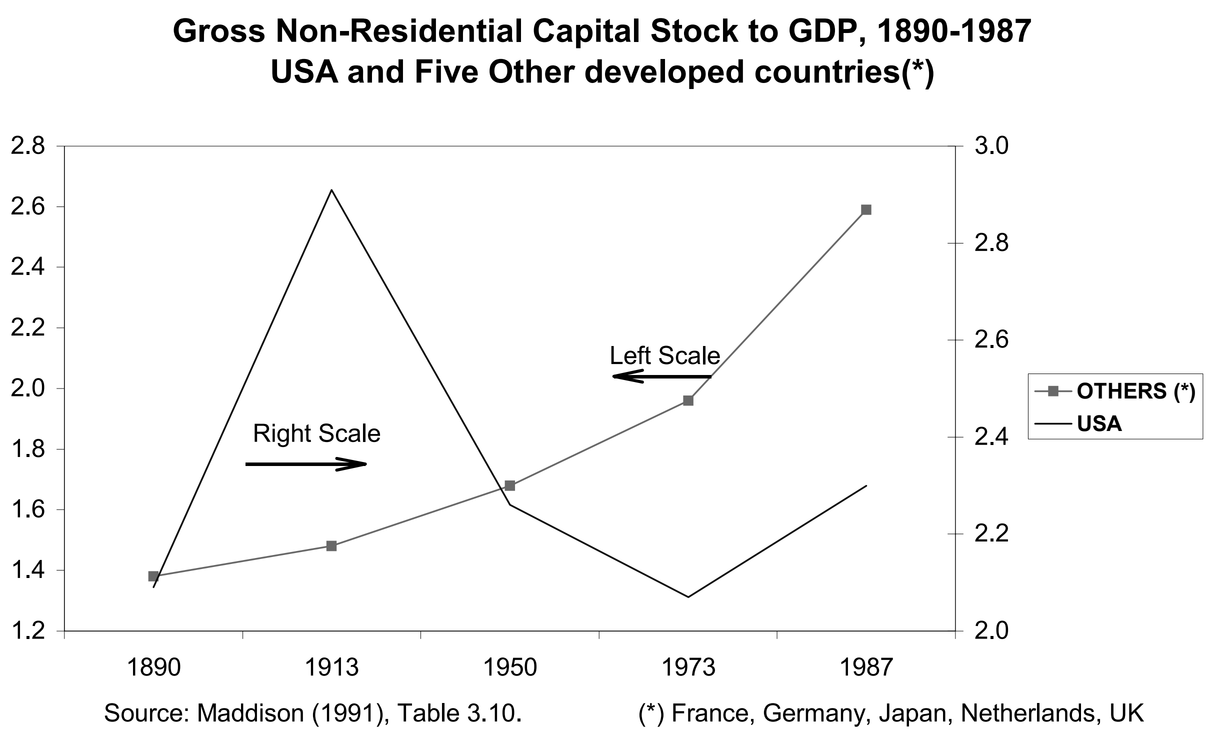

Figure 7: Capital-Output Ratio

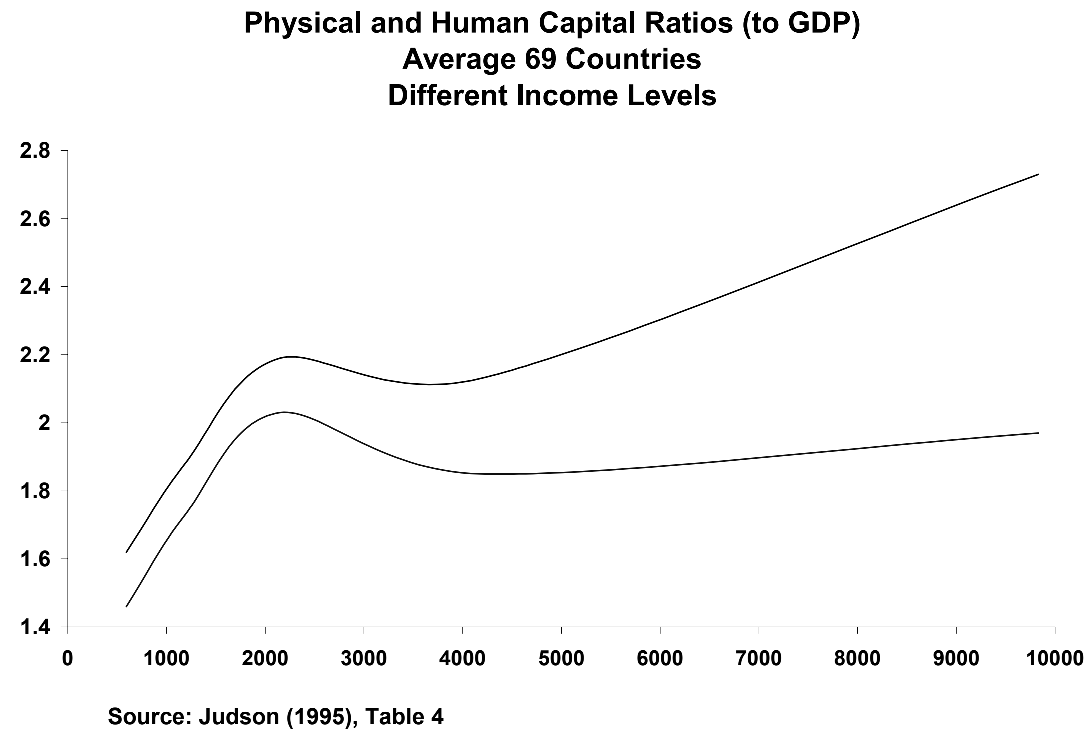

Figure 8: Capital-Output Ratio Vs Income Levels

The upward trend becomes sharper if human capital is included. In this respect, Judson (1995) has constructed different measures of total capital, including human and physical capital, for a cross section of countries and different periods, as shown in Figure 8. The upward trend is clear, a fact also stressed by Judson.

In conclusion, our simple model can readily explain why one usually observes Kaldor facts strongly associated with structural change regularities. In order to grow, countries must adopt new capital-intensive technologies, and cities provide advantages for the adoption of those technologies compared with rural places. Thus, growth requires urbanization.

5.3 Transient Growth: An Assessment

How much growth could be due to transitional dynamics? Early studies (e.g. King and Rebelo, (1993)) found that transitional growth can only play a minor role in explaining growth regularities because of the implausible changes required in the interest rate. To illustrate the point, consider a case where the only available technique is $F^2(K_t, A_tL_t) = K_t^\alpha(A_tL_t)^{1-\alpha}$ with $A_0 = 1$. Per-capita output is given by $y_t^p = A_tk_t^\alpha$ and the equilibrium interest rate satisfies $r_t + \delta = k_t^{\alpha-1}$. Let g be the growth rate of per-capita output between t_0 and t. Then, it follows that

$$g = \frac{A_t}{A_{t_0}}\left(\frac{r_t+\delta}{r_{t_0}+\delta}\right)^{\frac{\alpha}{\alpha-1}} \tag{16}$$

King and Rebelo seek to explain the sevenfold increase in U.S. per-capita output ($g = 7$). They allow technological progress to explain half of the observed growth ($\frac{A_t}{A_{t_0}} = \sqrt{7}$) and ask if the remaining growth can be explained by transitional dynamics, i.e., $\left(\frac{r_t+\delta}{r_{t_0}+\delta}\right)^{\frac{\alpha}{\alpha-1}} = \sqrt{7}$. For a standard value of $\alpha = 1/3$, the gross interest rate needs a sevenfold drop during the transition; for $\alpha = 1/2$, $\sqrt{7}$ times; and for $\alpha = 2/3$, $\sqrt[4]{7}$ times. Due to the lack of evidence supporting long term significant changes in the interest rates, King and Rebelo conclude that transitional dynamics could only play a minor role in the observed growth, particularly because α seems to be below 1/2.

We ask now the same question in our model. As we pointed out, transient growth can occur in our model without any change in the interest rate as the economy moves along the cone of diversification. According with Lemma 4, growth in the cone is equal to $g^C = \frac{1-\alpha(\underline{k})}{1-\alpha(\overline{k})}$. Suppose F^1 requires no capital and $F^2 = K_t^\alpha(A_tL_t)^{1-\alpha}$, as before. In that case, $\alpha(\underline{k}) = 0$, $\alpha(\overline{k}) = \alpha$, and $g^C = \frac{1}{1-\alpha}$. Then we require $g^C\left(\frac{r_t+\delta}{r_{t_0}+\delta}\right)^{\frac{\alpha}{\alpha-1}} = \sqrt{7}$ or $\left(\frac{r_t+\delta}{r_{t_0}+\delta}\right)^{\frac{\alpha}{\alpha-1}} = \sqrt{7}(1-\alpha)$. Thus, for $\alpha = 1/3$, the gross interest rate $(r+\delta)$ needs a $\frac{7}{2.25}$ drop instead of the sevenfold required before; for $\alpha = 1/2$, it needs to falls $\frac{\sqrt{7}}{2}$times instead of $\sqrt{7}$, and for $\alpha = 2/3$ it needs to fall $\frac{\sqrt[4]{7}}{1.73}$ times instead of $\sqrt[4]{7}$.

In conclusion, the required fall in the interest rate is less drastic in our model. The results also suggest that a large capital share is still needed if transitional dynamics is to be important for growth. Sensible results are obtained with $\alpha = 1/2$. This high value for α is plausible if human capital is included as part of capital. In any case, the role of human capital in our model needs not be as important as some studies suggest (for example, Barro *et. al.* (1995) pick α around 0.8).

5.4 Growth and Productivity Slowdown

A major puzzle in economic growth is the significant and persistent slowdown in growth experienced by most developed countries since early seventies. Although several explanations have been advanced, there is still no consensus about what caused the slowdown. Our model gives a natural explanation for the puzzle. It predicts that a *permanent* growth slowdown occurs when the economy surpasses the capital level $\bar{k}$. At that point the structural change is completed, capital accumulation in the advanced sector cannot be matched with labor coming from the backward sector so that the economy start facing decreasing marginal returns on capital. As a result, a growth slowdown occurs at the time when the economy reaches a steady level of urban population. According to the model, the growth slowdown is equal to the transitional growth as given by equation (11) evaluated at $k = \overline{k}$, i.e.,

$$\text{Growth Slowdown} = \frac{1}{\theta}\left(r(\overline{k}) - r(k^*)\right).$$

This prediction of the model is supported by evidence. According to Figure 2, the share of urban population increased steadily until the early seventies, when it suddenly almost stabilized. On average, it rose 0.32 points per year during the period 1960-75 but only 0.11 points per year during the period 1975-95. During this last period, most developed economies also experienced a major growth slowdown (See Maddison, 1991).

Inverse causality can be argued: the slowdown in income growth could explain the urbanization slowdown because of Engel's law. This is certainly a plausible argument, but it cannot be the whole story for at least three reasons. First, the urbanization slowdown was expected, regardless of the income trend, because the urban ratio was growing at an unsustainable rate given its upper bound at 1 (See Figure 2). Second, the evidence in Figure 1 shows no slowdown of the urban ratio in developing countries in spite of their dramatic income 'meltdown' during the eighties (Ben-David and Papell (1997)). This suggests that Engel's law is not the main determinant of urbanization. Finally, the urbanization slowdown seems too strong to be explained by Engel's law. Consider the case of the U.S. presented in Figure 9. Income remained increasing at a significant rate even after the slowdown while the urban ratio almost stopped growing.

An apparent drawback of this explanation are the findings in several studies indicating that slowdown is associated to a slowdown in productivity growth. There is no productivity slowdown in our model since productivity always grows at the constant rate x. It turns out, however, that the predictions of the model can be reconciled with the evidence of productivity. To see how, note that standard exercises compute series of productivity as Solow residuals. Those exercises typically assume that the aggregate production function is Cobb-Douglas with capital share around 1/3. If the true production

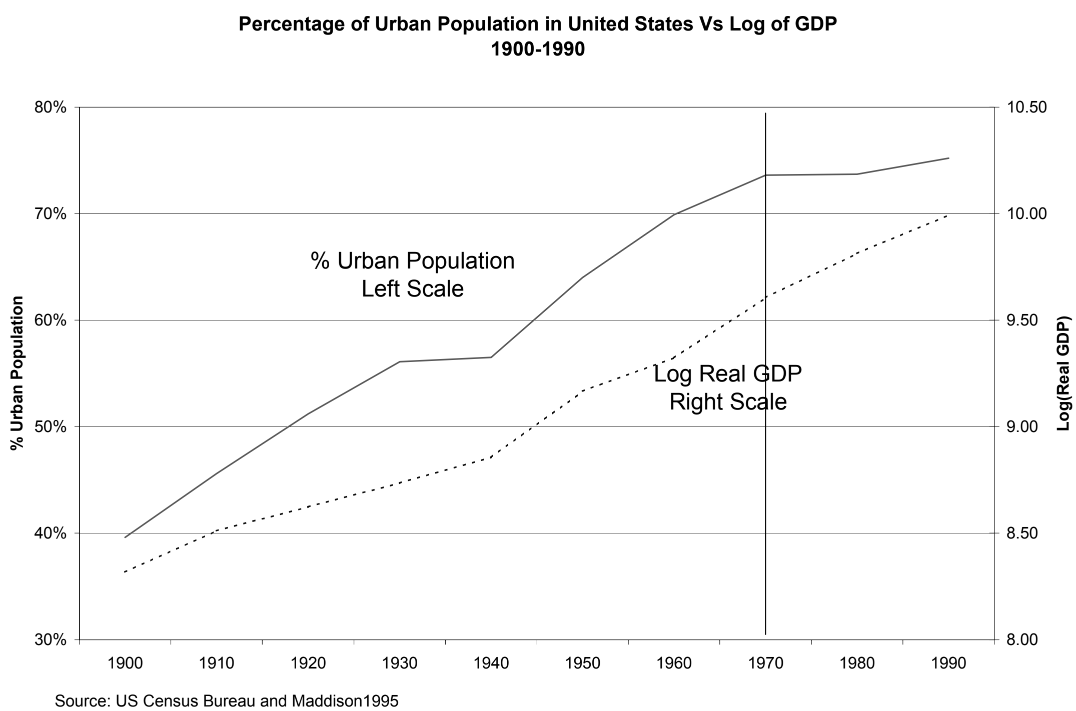

Figure 9: Urban Share Vs Percapita GDP

function is given by (5), then those exercises produce spurious series of productivity.

Let $\widehat{\alpha}$ be the estimated capital elasticity of output (usually 1/3), y^p per-capita income, and k^p per-capita capital stock. The Solow residuals (in levels) can be computed as

$$R_t = \frac{y_t^p}{(k_t^p)^{\widehat{\alpha}}} = \frac{y_t e^{xt}}{(k_t e^{xt})^{\widehat{\alpha}}} = \frac{y_t}{(k_t)^{\widehat{\alpha}}} e^{(1-\widehat{\alpha})xt}.$$

If the true production function is Cobb-Douglas with capital share $\widehat{\alpha}$, then Solow residuals are a correct measure of productivity: $R_t = e^{(1-\widehat{\alpha})xt}$. If, instead, the true production function is given by equation (5) and $\underline{k} = 0$ (to simplify), then the Solow residuals become

$$R(k_t, \widehat{\alpha}) = \left\{ \begin{array}{ll} (k_t)^{-\widehat{\alpha}} (A_1 + f_k^2(\overline{k}) \cdot k_t) e^{(1-\widehat{\alpha})xt} & \text{for } 0 \leq k_t \leq \overline{k} \\ (k_t)^{\alpha-\widehat{\alpha}} e^{(1-\widehat{\alpha})xt} & \text{for } k_t \geq \overline{k} \end{array} \right\},$$

and the growth rates of the residual are given by

$$\left(\frac{\partial}{\partial t} \log R\right)(k_t, \widehat{\alpha}) = \left\{ \begin{array}{ll} -\left(\widehat{\alpha} - f_k^2(\overline{k})\frac{k_t}{y_t}\right) \dot{k}_t + (1-\widehat{\alpha})x & \text{for } 0 \leq k_t \leq \overline{k} \\ (\alpha - \widehat{\alpha})\, \dot{k}_t + (1-\widehat{\alpha})x & \text{for } k_t \geq \overline{k} \end{array} \right\}. \quad (17)$$

Figure 10 illustrates this equation and the following lemma summarizes its properties. Note that $(1-\alpha)x$ is the rate of "Hicks neutral" technological progress.

- **Lemma 7** *(Growth of the Solow Residual) Let* $g^r(k_t, \widehat{\alpha}) \equiv \frac{\partial}{\partial t} \log R_t$. *Then, (i)* $g_t^r(k, \widehat{\alpha})$ *is continuous in k; (ii)* $g_t^r(k, \widehat{\alpha})$ *is increasing in k for* $0 \leq k_t \leq \overline{k}$; *(iii) given* $\widehat{\alpha}$ *and* $k_t \geq \bar{k}$, $g_t^r(k_t, \widehat{\alpha})$ *converges monotonically toward* $(1-\widehat{\alpha})x$; *(iv)* $g_t^r(\overline{k}, \widehat{\alpha}) > (1-\widehat{\alpha})x$ *for* $\widehat{\alpha} < \alpha$. *Also,* $g_t^r(\bar{k}, \widehat{\alpha})$ *is decreasing in* $\widehat{\alpha}$; *(v) if* $\widehat{\alpha} = \alpha$, $g_t^r(\overline{k}, \widehat{\alpha}) = (1-\widehat{\alpha})x$, *for* $k_t \geq \overline{k}$;

  Proof. Continuity in part (i) follows from the fact that $f_k^2(\overline{k})\frac{\bar{k}}{y} = \alpha$; $g_t^r(k_t)$ increasing for $0 \leq k_t \leq \overline{k}$ follows from the facts that k/y is strictly increasing (Proposition 2) and $\dot{k}_t$ is decreasing for that interval so that $\left(\widehat{\alpha} - f_k^2(\overline{k})\frac{k_t}{y_t}\right) \dot{k}_t$ is decreasing. The other properties follow from direct inspection of equation (17) and the fact that $\dot{k}_t$ monotonically converges toward zero. ■

From the lemma, it is clear that a "productivity slowdown" can be observed if the capital share on income, $\widehat{\alpha}$, is below the true capital elasticity of output, α. This is precisely a point made by

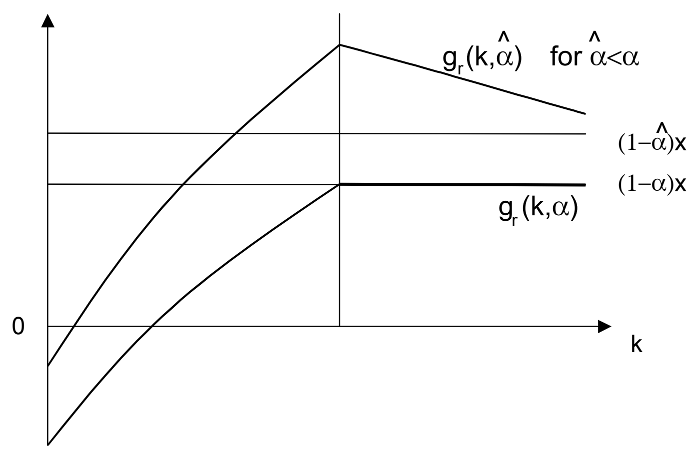

Figure 10: Productivity Slowdown

the recent growth literature. Several papers have suggested that α is larger than 0.5, if the role of human capital is considered (e.g. King and Rebelo (1990), Mankiw *et. al.* (1992), Barro and Sala-I-Marti (1990), among others). If that is the case, the model can easily explain a growth slowdown accompanied by a wrongly measured 'productivity slowdown'. The following proposition summarizes the discussion and follows from the last lemma.

Proposition 8 *("Productivity Slowdown"). Let $\widehat{\alpha} \leq \alpha$. Then $g^r(k_t, \widehat{\alpha})$ is strictly increasing for $0 \leq k_t \leq \overline{k}$ and decreasing for $k_t > \overline{k}$.*

6 Extensions

In this section we extend the model in two dimensions. First, we introduce land into the model and provide a justification for interpreting labor in the advanced sector as urban labor. According with this explanation, worker in the advanced sector agglomerates around the most productive piece of land. The second extension is design to account for another feature of the structural change: the change in the composition of output. For this purpose we introduce agricultural and non-agricultural goods into the model. We show that the extended model can account for changes in the composition of output, and for the decay of the agricultural labor share even under homothetic preferences.

6.1 Land and Agglomeration

Given the set up of the model, we would like to interpret l_1 as the share of rural population, and l_2 as the share of urban population. For that purpose, we introduce space and induce agglomeration in our model in a way that keeps the results of the previous section unaffected.

Suppose land is homogeneously spread along a straight line of length 1, although any other spatial configuration is equally useful. Locations are denoted by j, for $j \in [0, 1]$ and each location has one unit of land. Land is another factor of production, homogenous from the perspective of the backward sector (for example, all land is equally fertile), but heterogeneous from the perspective of the advanced one. In particular, land productivity is highest at location j^* (due, for example, to the existence of a port or natural resource there) and it smoothly decreases with the distance from j^*. Thus, efficiency dictates that the production of the advanced sector must agglomerate around j^*.

More precisely, suppose the backward technique requires one unit of land per worker, i.e., $F^1(K_1, A_1(L_1, T_1))$ $= F^1(K_1, A_1 \min\{L_1, T_1\})$, where T_1 stands for land. Assume also that there is enough land so that labor is the limiting factor. In that case, $F^1(K_1, A_1L_1, T_1) = F^1(K_1, A_1L_1)$ as before. In this case labor in the backward sector can be considered rural since it is spread along the line.

The advanced technique, on the other hand, requires $\epsilon < 1$ units of land per worker. Production at location j with the advanced technique is given by $F^2(K_2^j, A_2^j(L_2^j, T(j))) = F^2(K_2^j, A_2^j \min\left\{L_2^j, 1/\epsilon\right\})$. For ϵ sufficiently close to 0, all labor in the advanced sector agglomerates at j^*. Total advanced production is $F^2(K_2^{j^*}, A_2^{j^*}L_2^{j^*})$, as in the previous section. The advantage of this formulation is that we can now interpret labor in the advanced sector as urban labor because it agglomerates at a single location.

This approach to agglomeration retains the results of the previous section and preserve the efficiency of the competitive equilibrium. It is supported by the evidence that most cities are located in places with particular geographic advantages, such as coasts, rivers, minerals resources or fertile soil (e.g. Fujita *et al* (1999)).

An alternative to assume a best location is to allow for externalities in the advanced sector, as in Henderson (1974). According to this approach, factors are more productive working together than apart. The competitive equilibrium in this case is not efficient in general; but the qualitative results obtained above can still apply. To see this, suppose F^2 is given by $F^2(K_2^j, K_2^{A,j}, L_2^j)$, where K_2^j is individual capital, $K_2^{A,j}$ is aggregate capital in the advanced sector al location j, and F^2 is linear homogenous in its three arguments. A positive externality is introduced by making F^2 increasing in K_2^A. In a competitive equilibrium, marginal products must be equal as long as techniques are combined, i.e.

$$\begin{aligned} F_l^1(k_{1t}, 1) &= F_l^2(k_{2t}, k_{2t}^A, 1) \\ F_k^1(k_{1t}, 1) &= F_k^2(k_{2t}, k_{2t}^A, 1), \end{aligned}$$

where k_1, k_2 and k_2^A are capital-labor ratios in each technique, and F_k^2 is the partial derivative of F^2 with respect to private capital. Aggregate constraints also impose $k_{2t} = k_{2t}^A$ as all firms using F^2 locate together because of the externality[10]. The two previous equalities thus form a system of two equations in two unknowns, k_1 and k_2, that can be solved independently of the aggregate capital-labor ratio. This is the mechanism that gives rise to a production function with linear segments, as

[10] This is not the only possible equilibrium agglomeration; but it is the only one robust to sensible refinements.

shown in the previous section.

6.2 Multiple goods

We can extent the model to capture changes in the composition of output, and to allow for non-zero rural population in steady state. In particular, we aim to capture an increasing share of manufactured output and a decreasing share of agriculture output. To reduce notation suppose there is no exogenous technological progress, no population growth and total labor equals 1. Suppose there are two essential goods in the economy: an agricultural good and a manufactured good. The manufacturing sector in this model supplies consumption and investment goods, while the agricultural sector only supplies consumer goods. A convenient but not essential assumption is to suppose that both goods can be produced with exactly the same techniques. Let $F^1(K,L) = A_1 L$ and $F^2(K,L) = A_2 K^\alpha L^{1-\alpha}$. Since both goods can be produced with the same techniques then the relative price of the goods is 1[11]. Let c_A and c_M be the consumption of agricultural and manufactured good respectively and let the instantaneous utility function be given by

$$u(c_A, c_M) = \frac{1}{1-\sigma}\left((c_A - \underline{c})^{1-\gamma} c_M^\gamma\right)^{1-\sigma},$$

where $\underline{c}$ is a minimum consumption requirement and it satisfies $A_1 > \underline{c} \geqq 0$. Equating the relative price to the marginal rate of substitution we have:

$$1 = \frac{u_{c_A}(c_A, c_M)}{u_{c_M}(c_A, c_M)} = \frac{1-\gamma}{\gamma}\frac{c_M}{c_A - \underline{c}} \tag{18}$$

Let l_a be the share of total labor in agriculture. We want to see what happen to l_a as the economy grows. Before the industrial revolution, $c_A = A_1 l_a$ and $c_M = A_1(1 - l_a)$. Replacing these two equalities into (20) and solving from l_a we get

$$l_a = 1 - \gamma + \frac{\gamma \underline{c}}{A_1}.$$

Now consider an steady state of this economy in which only F^2 is used. Such steady state exist if and only if $k^* > \overline{k}$ where $\overline{k} = \left(\frac{A_1}{(1-\alpha)A_2}\right)^{\frac{1}{\alpha}}$. Efficiency implies that the same capital/labor ratio, k^*, is used in both sectors. Note also that k^* is the aggregate level of capital. Therefore, $c_A = A_2 k^{*\alpha} l_a^*$ and $c_M = A_2 k^{*\alpha}(1 - l_a^*) - \delta k^*$. Replacing the two last expressions into (18) and solving for l_a^* :

[11] This fact can be seen by equating marginal products of capital and labor across sectors.

$$\begin{aligned} l_a^* &= (1-\gamma)\left(1-\frac{\delta k^*}{A_2 k^{*\alpha}}\right)+\gamma\frac{\underline{c}}{A_2 k^{*\alpha}} \\ &< (1-\gamma)+\gamma\frac{\underline{c}}{A_2 k^{*\alpha}} \\ &< (1-\gamma)+\gamma\frac{\underline{c}}{A_1}=l_a \end{aligned}$$

The last inequality follow from requiring $k^* > \overline{k}$ so that the advanced technique is adopted. In that case $A_2 k^{*\alpha} > A_2 \overline{k}^{\alpha} = \frac{A_1}{(1-\alpha)} > A_1$. Therefore, economic growth in this model result in a lower the labor share in agriculture and a larger labor share in the manufacturing sector. This is true even if $\underline{c} = 0$, i.e., even with homothetic preferences. The recomposition of output in this case is driven by the fact that capital is a manufactured good. In this last case. $l_a - l_a^* = (1-\gamma)\frac{\delta k^*}{A_2 k^{*\alpha}} = (1-\gamma)\,\delta\frac{k^*}{y^*}$. Suppose large values for $\frac{k^*}{y^*}, \delta$ and γ so that we can find an upper bound for $l_a - l_a^*$. Let $\frac{k^*}{y^*} = 4$, $\delta = 10\%$ and $\gamma = 0.5$. Then, $l_a - l_a^* = 0.2$. These computations suggest that a significant amount of structural change can be explained with homothetic preferences. But it also suggest that some non-homotheticity is required.

7 Final Remarks

We show that capital-biased technological inventions can induce adjustment dynamics that are surprisingly consistent with very diverse and crucial regularities of economic growth. First, it is consistent with most of the Kaldor facts: the interest rate, the growth, and the labor share are constant, and the capital-labor ratio is increasing along the adjustment path. Second, the transition displays a gradual reallocation of labor from the backward sector to the advanced one, which is consistent with cross-country evidence about urbanization and structural change. Third, the model predicts that a growth slowdown takes place once the structural change is completed, a prediction consistent with important evidence about growth, productivity slowdown, and urban shares. Four, the model can also explain Kuznets curve (Glachant (2000)). Five, the model predicts that the capital-output ratio increases, a prediction that conflicts with one of the Kaldor facts but that is strongly supported by the evidence. Finally, the model provides an important amplification mechanism: small changes in preferences or technologies can induce large differences in per-capita income.

In our opinion, these features of the model make a good case for the use of capital-biased inventions of the type considered here in growth models. These type inventions induce a more realistic transitional dynamics underlying balanced growth paths generated by labor-augmenting inventions. We also regard our explanation for the structural change as complementary to the more traditional explanation based on Engel's law. We think that in order to account for the main features of the data, preferences may not be homothetic, and technologies may not be strictly concave. Lastly, our approach provides microfoundations for the use of $AK + BL$ technologies in growth models. They naturally arise when technological progress is dramatic and biased.

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9 Appendix 1.1

We prove Proposition 1 through a series of lemmata. To reduce notation, assume no technological progress nor population growth and $A_1 = A_2 = 1$. Define $G : [0, K] \times [0, L] \to \Re^+$ by

$$G(K_1, L_1; K, L) \equiv F^1(K_1, L_1) + F^2(K - K_1, L - L_1)$$

Let $D(K, L)$ be the domain of $G : D(K, L) = [0, K] \times [0, L]$. Define H as the arg max of G:

$$H(K, L) = \left\{ \begin{array}{c} (K_1^*, L_1^*) : G(K_1^*, L_1^*) \geq G(K_1, L_1) \\ \text{for } \forall (K_1, L_1) \in D(K, L) \end{array} \right\}$$

G and D satisfy the assumptions of Weierstrass Theorem so that $H(K, L)$ is not empty. Note that $F(K, L) = \max_{0 \leq K_1 \leq K, 0 \leq L_1 \leq L} G(K_1, L_1; K, L)$. Let $MRTS^i(k) = \frac{F_L^i(k,1)}{F_K^i(k,1)}$ be the marginal rate of technical substitution for technique $i = 1, 2$.

Lemma 9 *Let $K/L = \widetilde{k}$. Then, (i) $F_L^1(K, L) > F_L^2(K, L)$, $F_K^2(K, L) > F_K^1(K, L)$ and $MRTS^1(K/L) > MRTS^2(K/L)$; (ii) $F(K, L) > F^1(K, L)$ and $F(K, L) > F^2(K, L)$ (it is optimal to use both techniques).*

Proof. (i) From Assumption 1.2, $F^2(K + \Delta K, L) > F^1(K + \Delta K, L)$ for $\Delta K > 0$. Since the $F^i()$'s are linear homogenous and differentiable, it follows that $F_K^2(\widetilde{k}, 1) = F_K^2(K, L) > F_K^1(K, L) = F_K^1(\widetilde{k}, 1)$. In the same vein, $F_L^1(\widetilde{k}, 1) = F_L^1(K, L) > F_L^2(K, L) = F_L^2(\widetilde{k}, 1)$.

(ii) By contradiction, suppose $F(K, L) = F^1(K, L)$ (a similar argument holds for $F(K, L) = F^2(K, L)$). Take from F^1 factors $\Delta K (> 0)$ and $\Delta L (> 0)$ satisfying $\frac{\Delta K}{\Delta L} > \frac{F_L^1(\widetilde{k},1) - F_L^2(\widetilde{k},1)}{F_K^2(\widetilde{k},1) - F_K^1(\widetilde{k},1)} > 0$ and transfer them into F^2. The last inequality follows from part (i). Then, for ΔK and ΔL sufficiently small, the change in total output is given by $\left(F_K^2(\widetilde{k}, 1) - F_K^1(\widetilde{k}, 1)\right) \Delta K + \left(F_L^2(\widetilde{k}, 1) - F_L^1(\widetilde{k}, 1)\right) \Delta L > \left(F_L^1(\widetilde{k}, 1) - F_L^2(\widetilde{k}, 1) + F_L^2(\widetilde{k}, 1) - F_L^1(\widetilde{k}, 1)\right) \Delta L = 0$. Then F^1 is not optimal. A contradiction. ■

Lemma 10 *Let $(K_1^*, L_1^*) \in H(K, L)$. (i) If $L > L_1^* > 0$ then $\frac{K_1^*}{L_1^*} < \widetilde{k} < \frac{K - K_1^*}{L - L_1^*}$. (ii) If $L_1^* = 0$, then $\frac{K}{L} > \widetilde{k}$. (iii) If $L_1^* = L$, then $\frac{K}{L} < \widetilde{k}$*

Proof. (i) By contradiction suppose $\frac{K_1^*}{L_1^*} \geq \widetilde{k}$. Then, by Assumption 1.2, $F^2(K_1^*, L_1^*) \geq F^1(K_1^*, L_1^*)$. If $F^2(K_1^*, L_1^*) > F^1(K_1^*, L_1^*)$ then (K_1^*, L_1^*) is not optimal (a contradiction). If $F^2(K_1^*, L_1^*) = F^1(K_1^*, L_1^*)$

then $\frac{K_1^*}{L_1^*} = \widetilde{k}$. But from Lemma 9 $F(K_1^*, L_1^*) > F^1(K_1^*, L_1^*)$ (a contradiction). A similar argument can be used to prove that $\widetilde{k} < \frac{K-K_1^*}{L-L_1^*}$.

(ii) Suppose, by contradiction, that $\frac{K}{L} \leq \widetilde{k}$. Since $L_1^* = 0$ then $K_1^* = 0$ and $F(K,N) = F^2(K,N)$. However, by Assumption 1.2, $F^2(K,N) \leq F^1(K,N)$. If the inequality is strict then $L_1^* = 0$ and $K_1^* = 0$ is not optimal (a contradiction). If a equality prevail, then $K/N = \widetilde{k}$ so that by Lemma 9 $F(K,N) > F^2(K,N)$ (A contradiction).

(iii) Similar to part (ii). ■

Lemma 11 *$G(K_1, L_1; K, L)$ is concave in (K_1, L_1). It is strictly concave in $(K_1, L_1) \in intD(K,L)$ for $K_1/L_1 \neq (K - K_1)/(L - L_1)$.*

Proof. The Hessian associated to G is given by

$$H = \begin{bmatrix} F_{KK}^1 + F_{KK}^2 & F_{KL}^1 + F_{KL}^2 \\ F_{LK}^1 + F_{LK}^2 & F_{LL}^1 + F_{LL}^2 \end{bmatrix}$$

where $F_{XY}^i = \frac{\partial}{\partial Y}\left(\frac{\partial F^i(X,Y)}{\partial X}\right)$. The terms in the diagonal are strictly negative by Assumption 1.1. Thus, G is concave if $|H| \geq 0$ and strictly concave if $|H| > 0$. By Young's Theorem and the fact that G is C^2, $|H| = \left(F_{KK}^1 + F_{KK}^2\right)\left(F_{LL}^1 + F_{LL}^2\right) - \left(F_{KL}^1 + F_{KL}^2\right)^2$. Linear homogenous functions have the property that $F_{KK}^i F_{LL}^i - \left(F_{KL}^i\right)^2 = 0$. Therefore, $|H| = F_{KK}^1 F_{LL}^2 + F_{KK}^2 F_{LL}^1 - 2F_{KL}^1 F_{KL}^2$. Other properties of linear homogenous functions are $F_{LL}^i = \left(\frac{K_i}{L_i}\right)^2 F_{KK}^i$ and $F_{KL}^i = -\frac{K_i}{L_i} F_{KK}^i$. Using these two properties into the previous expression we get

$$\begin{aligned} |H| &= F_{KK}^1 \left(\frac{K_2}{L_2}\right)^2 F_{KK}^2 + F_{KK}^2 \left(\frac{K_1}{L_1}\right)^2 F_{KK}^1 - 2\frac{K_1}{L_1}\frac{K_2}{L_2} F_{KK}^1 F_{KK}^2 \\ &= F_{KK}^1 F_{KK}^2 \left[\left(\frac{K_2}{L_2}\right)^2 + \left(\frac{K_1}{L_1}\right)^2 - 2\frac{K_1}{L_1}\frac{K_2}{L_2}\right] \\ &= F_{KK}^1 F_{KK}^2 \left[\left(\frac{K_2}{L_2}\right) - \left(\frac{K_1}{L_1}\right)\right]^2 > 0 \text{ for } \left(\frac{K_2}{L_2}\right) \neq \left(\frac{K_1}{L_1}\right) \end{aligned}$$

■

Lemma 12 *$H(K,N)$ is singlenton.*

Proof. Let $(K_1^*, L_1^*) \in H(K,L)$. (i) If $L > L_1^* > 0$ then $\frac{K_1^*}{L_1^*} \neq \frac{K-K_1^*}{L-L_1^*}$ from Lemma 10. Using this fact and the previous Lemma 11, it follows that (K_1^*, L_1^*) is a unique local maximizer. Since G is concave it also follows that (K_1^*, L_1^*) is the unique global maximizer. (ii) If $L_1^* = 0$, then

$F(K,N) = F^2(K,N)$. This must be the only solution. Suppose not. Then $\exists\ (K_1^{*\prime}, L_1^{*\prime}) \neq (K_1^*, L_1^*)$ such that $F(K,N) = G(K_1^{*\prime}, L_1^{*\prime}) = G(K_1^*, L_1^*)$. From the previous result $L_1^{*\prime} \notin (0,L)$. Otherwise $(K_1^{*\prime}, L_1^{*\prime})$ would be the unique maximizer. Then there are two alternatives (a) $K_1^{*\prime} = K$ and $L_1^{*\prime} \in \{0, L\}$ which implies $L_1^{*\prime} = L$; (b) $0 \leq K_1^{*\prime} \leq K$ and $L_1^{*\prime} = L$ which implies $K_1^{*\prime} = K$; In both cases it follows that $G(K_1^{*\prime}, L_1^{*\prime}) = F^1(K,L) = F^2(K,L) = F(K,L)$. A contradiction. ■

Let $K_1(K,N)$ be the first component of $H(K,N)$ and $L_1(K,N)$ the second component. Define $K_2(K,N) \equiv K - K_1(K,N)$ and $L_2(K,N) \equiv L - L_1(K,N)$. When (K,N) is well defined by the context we just write K_1^* instead of $K_1(K,N)$, etc.

Define

$$\begin{aligned} \underline{k} &= \frac{K_1(\widetilde{k},1)}{L_1(\widetilde{k},1)} \\ \overline{k} &= \frac{K_2(\widetilde{k},1)}{L_2(\widetilde{k},1)} \end{aligned}$$

$\underline{k}$ and $\overline{k}$ are well defined since $L > L_1(\widetilde{k},1) > 0$ (Lemma 9). Also note that $\underline{k}$ can be equal to zero due to the fact that capital is not essential to produce with technique 1.

The following result follows from Lemma 9 and efficiency considerations:

Lemma 13 *(i)* $\underline{k} < \overline{k}$ *(ii)* $F_L^1(\underline{k},1) = F_L^2(\overline{k},1)$ *and* $F_K^1(\underline{k},1) \leq F_K^2(\overline{k},1)$ *with equality if* $\underline{k} > 0$.

Proof. (i) Follows directly from the fact that $\underline{k} < \widetilde{k} < \overline{k}$ and (ii) follow from the fact that the solution is interior. (The marginal product of labor across techniques must be equal since labor is essencial to produce with either technique. The marginal product of capital need not to be equal in an efficient allocation since capital is not essencial to produce with the TT technique.) ■

Lemma 14 *Let* $\frac{K}{L}$ *be such that* $\overline{k} > \frac{K}{L} > \underline{k}$. *Then, (i)* $\frac{K_1(K,L)}{L_1(K,L)} = \underline{k}$, $\frac{K_2(K,L)}{L_2(K,L)} = \overline{k}$, *(ii)* $L_1(K,L) = \left(\frac{\overline{k} - K/L}{\overline{k} - \underline{k}}\right) L$ *and* $L_2(K,L) = \left(\frac{K/L - \underline{k}}{\overline{k} - \underline{k}}\right) L$, *(iii)* $F(K,L) = F_K^2(\overline{k},1)K + F_L^2(\overline{k},1)L$

Proof. (i) We just need to check that the proposed allocation satisfies the Kunh-Tucker first-order conditions (Theorem 7.16, Sundaram). Sufficiency of the first order conditions is assured by concavity of G. Uniqueness is assured by strict concavity of G at the proposed allocation. The K-T first-order conditions of the problem under the proposed allocation are:

(a) $F_L^1(\frac{K_1(K,L)}{L_1(K,L)},1) = F_L^2(\frac{K_2(K,L)}{L_2(K,L)},1)$

(b) $F_K^1(\frac{K_1(K,L)}{L_1(K,L)},1) \leq F_K^2(\frac{K_2(K,L)}{L_2(K,L)},1)$ with equality if $K_1(K,L) > 0$.

(c) $K = K_1(K,L) + K_2(K,L)$

(d) $L = L_1(K,L) + L_2(K,L)$

By construction, $\frac{K_1(K,L)}{L_1(K,L)} = \underline{k}$ and $\frac{K_2(K,L)}{L_2(K,L)} = \overline{k}$ satisfy (a) and (b) since $\underline{k}$ and $\overline{k}$ are the solutions for $K/L = \widetilde{k}$. Note also that $K_1(K,L) + K_2(K,L) = \underline{k}L_1(K,L) + \overline{k}L_2(K,L) = \underline{k}(1 - \frac{K/L-\underline{k}}{\overline{k}-\underline{k}})L + \overline{k}\frac{K/L-\underline{k}}{\overline{k}-\underline{k}}L = K$ and $L_1 + L_2 = L$.

(ii) $K = K_1(K,L) + K_2(K,L) = \underline{k}L_1(K,L) + \overline{k}(L - L_1(K,L))$. Then $L_1(K,L) = \frac{\overline{k}-K/L}{(\overline{k}-\underline{k})}L$

(iii)$F(K,L) = F^1(\underline{k},1)L_1(K,L) + F^2(\overline{k},1)(L - L_1(K,L)) = \left(F^1(\underline{k},1) - F^2(\overline{k},1)\right)L_1(K,L) + F^2(\overline{k},1)L =$

$\left(F_K^1(\underline{k},1)\underline{k} - F_K^2(\overline{k},1)\overline{k}\right)L_1(K,L) + F^2(\overline{k},1)L =$

$\left(F_K^1(\underline{k},1)\underline{k} - F_K^2(\overline{k},1)\overline{k}\right)\frac{\overline{k}-K/L}{(\overline{k}-\underline{k})}L + F^2(\overline{k},1)L =$

There are two cases: (a) $\underline{k} > 0$. Then $F_K^1(\underline{k},1) = F_K^2(\overline{k},1)$ so that $F(K,L) = F_K^2(\overline{k},1)K + \left(F^2(\overline{k},1) - F_K^2(\overline{k},1)\overline{k}\right)L$;(b) $\underline{k} = 0$. Then the same result follows. ■

Lemma 15 *(i)* $F(K,L) = F^1(K,L)$ *for* $K/L \leq \underline{k}$ *(ii)* $F(K,L) = F^2(K,L)$ *for* $K/L \geq \overline{k}$.

Proof. We only prove part (ii). To prove part (i) similar arguments can be used. Note that $\frac{K_2^*}{L_2^*} \geq \overline{k}$. Otherwise $F(K_2^*,L_2^*) > F^2(K_2^*,L_2^*)$ so that (K_2^*,L_2^*) would not be optimal. Suppose by contradiction (of ii), that $L_1^* > 0$. Then $F_L^1(\frac{K_1^*}{L_1^*},1) = F_L^2(\frac{K_2^*}{L_2^*},1)$. If $\frac{K_2^*}{L_2^*} = \overline{k}$ then $\frac{K_1^*}{L_1^*} = \underline{k}$ but then $(K_1 + K_2)/L = \underline{k}\frac{L_1}{L} + \overline{k}\frac{L_2}{L} \leq \overline{k} < K/L$ so that there are unemployed resources. This cannot be optimal. Therefore, $\frac{K_2^*}{L_2^*} > \overline{k}$. Since marginal products of labor need to be equal, it follow that $\widetilde{k} > \frac{K_1^*}{L_1^*} > \underline{k}$, and from Lemma 14 $F(K_1^*,L_1^*) > F^1(K_1^*,L_1^*)$ so that (K_1^*,L_1^*) is not optimal. Therefore, (ii) must be true. ■