Macroeconomic Fluctuations in the United States: Demand or Supply, Permanent or Temporary?

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Abstract

We use generalized method of moments to estimate a rational expectations aggregate demand/aggregate supply macroeconomic model for the US economy. Variants of the model have been extensively used in analyses of optimal monetary policy under rational expectations. Our aim is to examine whether supply or demand shocks have predominated in the post-war era, and whether shocks of either type have been primarily temporary or permanent in nature. The estimation procedure is an alternative to estimating and interpreting vector autoregressions under restrictions either of the Bernanke-Sims variety or of the Blanchard-Quah variety or to performing calibration exercises.

We find that permanent nominal demand shocks are the predominant source of variance in output growth in the U.S., while permanent supply shocks are the main source of inflation variance and longer run autocorrelation of, and cross correlations between, output growth, inflation and interest rate changes. Temporary real shocks to demand are sizeable, and particularly important as a source of variance for changes in interest rates.

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1. Introduction

In recent years a number of authors have used vector autoregressions (VAR's) or calibration exercises to investigate whether macroeconomic fluctuations are primarily caused by nominal or real shocks. We contribute to this literature by estimating an aggregate demand/aggregate supply model with rational expectations that allows for both permanent and temporary shocks to both demand and supply.

The model we examine was suggested by Rogoff (1985) and has been used extensively in analyses of optimal monetary policy. A distinctive feature of our analysis is that we allow the shocks to aggregate demand and supply to have both permanent and temporary components that may not be separately identifiable. In other words, the number of driving shocks exceeds the number of endogenous variables. Nevertheless, we are able to estimate the structural parameters, including the variances of the underlying shocks, using generalized method of moments. Allowing for expectations and lags, and permanent and temporary components of each type of shock, enables our model to account for more complicated patterns of correlations between prices and output than are usually considered in studies of the sources of macroeconomic fluctuations.

Initially, we estimate a model of output and prices that combines real and nominal shocks to demand into a single demand shock that has permanent and temporary components. The results indicate that the permanent component is essentially nominal in character. We then add a third variable, the interest rate, that makes it possible to break down the overall demand shock into its real and nominal components and to estimate the parameters of an activist monetary policy.

The results consistently show that permanent nominal demand shocks are the largest single source of variance in output growth for the United States, followed by permanent supply shocks. Temporary real shocks to demand play a significant role in generating interest-rate movements, and it appears that monetary policy tends to accommodate such shocks.

Many other papers have investigated these issues. The classic paper by Sims (1980) found that nominal shocks were a major source of U.S. fluctuations. Sims argued that the exclusion restrictions commonly used to identify parameters in traditional structural models were not reasonable under rational expectations. When expectations are rational, all relevant predictive variables belong in any equation where expectations appear. While a VAR treats all observable variables as endogenous, the parameter estimates are very difficult to interpret. As a substitute for exclusion restrictions, Sims assumed that his data could be ordered in a Wold causal chain. Since then, various other methods of identifying VAR's have been proposed.

Blanchard and Watson (1986) identify a VAR by restricting the contemporaneous correlations of the one-step-ahead forecast errors. They conclude that U.S. fluctuations are due to fiscal, monetary, demand, and supply shocks, in roughly equal proportions.

Several other authors have used long-run restrictions to identify VAR's. After assuming that demand shocks have zero long-run impact on output, Blanchard and Quah (1989) find that demand shocks are the primary source of U.S. fluctuations. By contrast, Shapiro and Watson (1988) find evidence that exogenous labor supply shocks drive U.S. fluctuations. King, Plosser, Stock, and Watson (1991), who use a combination of long and short-run restrictions to identify their VAR's, report that nominal shocks have little importance and find evidence of at least two separate real shocks.

Gali (1992) examines a structural VAR of the IS-LM variety for the U.S. economy. He assumes there are four shocks: supply, money demand, money supply, and an IS shock (that is, three types of "demand" shocks, and one supply/productivity shock). He identifies parameters through a combination of long-run and short-run restrictions. He finds both types of shocks important, but supply shocks are dominant: 70 percent of output variability at business cycle frequencies is accounted for by supply shocks.

In this paper, we also estimate a small structural model of fluctuations of output, inflation, and interest rates in the U.S. We follow Hartley and Walsh (1992), however, and use a method of moments procedure to identify the parameters rather than long-run restrictions of the Blanchard-Quah variety. Our results thus are immune from the Lippi and Reichlin (1993) and Faust and Leeper (1994) criticisms of the Blanchard-Quah approach. In addition, structural modeling of the type pursued in this paper gives the estimated parameters a clear economic interpretation, something often lacking in VAR analyses.

Because the number of unobserved exogenous shocks in our model exceeds the number of observed endogenous variables, we cannot recover a time series for the shocks from the data. However, the endogenous variables can be expressed as a vector autoregressive moving average process of the shocks. This VARMA representation yields expressions for the contemporaneous and lagged variance and covariances of the endogenous variables as a function of the various supply and demand elasticities and the variances of the underlying shocks.

An initial estimation chooses parameter values to minimize the sum of squared differences between the theoretical second moments and the corresponding sample second moments obtained from the data. A second estimation minimizes a weighted sum of squared deviations with weights chosen "optimally" to yield a test of the parameter restrictions.

The method of moments estimation we use also is closely related to the "calibration method" often used to evaluate real business cycle models such as those pioneered by Kydland and Prescott (1982) and Long and Plosser (1983). Whereas the parameters are usually at best just-identified in the typical calibration exercise, however, the number of moments fit in the method of moments estimation can exceed the number of parameters. The over-identifying restrictions can then be tested. The method of moments procedure also allows us to estimate standard errors for the parameter values and this provides further information on the fit between the model and the data.[1]

2. Integration and co-integration tests

There are few a priori theoretical restrictions on the possible number, or stationarity properties, of the shocks affecting the macroeconomy. Before developing and estimating the model, therefore, the data need to be examined for stationarity and possible co-integration features. The assumed stochastic structure of the theoretical model then needs to be consistent with the stationarity properties of the data.

Quarterly data on industrial production and producer prices, both seasonally-adjusted, were obtained from Haver. We also used the interest rate on 3-month Treasury bills as the representative interest rate. We chose industrial production rather than GDP to avoid the problems with measuring government output that infect the GDP statistics. As for prices, we reasoned that industrial output responds more directly to producer prices than to other indices such as consumer prices. Furthermore, a price index may be preferable to an implicit deflator for our purpose since the latter may be negatively correlated with measures of real activity by construction.

In order to assess the number of unit roots (permanent shocks) in the data, two tests for unit roots and stationarity were used: the well-known augmented Dickey-Fuller test, which takes the presence of a unit root as the null hypothesis and stationarity as the alternative, and the test developed by Kwiatkowski, Phillips, Schmidt and Shin (1992) (KPSS), which takes stationarity as the null. For industrial production and producer prices, the tests were applied to the level of each series, logged, as well as to their growth rates (the first difference of the logs). For the interest rate, the tests were applied to the level of the series and the first difference.[2] The results are shown in Table 1.

The results are clearest in the case of interest rates: in levels, the KPSS test rejects trend stationarity, and

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1. Another difference is that each of the moments matched in a typical calibration exercise depends on a small number of parameters. We examine a full set of second moments, each of which is a complicated function of the parameters. It is no longer obvious how parameter values should be set so as to optimally match the theoretical and sample second moments.

2. Because there were signs of trends in the series, especially in industrial production and producer prices, all the test regressions on the levels included a linear trend as well as a constant term. Power considerations led us to include only a constant term in the test regressions on growth rates and first differences.

the Dickey-Fuller test fails to reject a unit root, indicating the presence of at least one unit root. Once the interest rate data have been first-differenced, the KPSS test fails to reject stationarity, and the Dickey-Fuller test rejects the presence of a second unit root at the 5 and even the 1 percent level.

TABLE 1. Tests for unit roots and stationarity[a]

	Industrial Production		Producer Prices		Interest Rate	
	Level	Growth Rate	Level	Growth Rate	Level	First Difference
D-F	-3.65*	-4.93*	-1.39	-2.58	-2.02	-5.26*
	(-3.44)	(-2.88)	(-3.44)	(-2.88)	(-3.44)	(-2.88)
KPSS	0.425*	0.102	0.422*	0.417	0.470*	0.077
	(0.146)	(0.463)	(0.146)	(0.463)	(0.146)	(0.463)

a. The 5 percent critical value for each test is in parentheses below the test statistic; those that are significant at the 5 percent level are marked with an asterisk. The augmented Dickey-Fuller tests were done including zero to nine lags of the dependent variables in order to deal with the possibility of serially-correlated residuals. The ones shown in the table were chosen using the suggestion in Campbell and Perron (1991, p 155). The KPSS tests included four lags to deal with the possibility of serially-correlated errors.

The results for the other variables are less definite. In the case of the level of industrial production, the two tests seem contradictory: the KPSS test rejects stationarity, but the Dickey-Fuller test rejects the presence of a unit root at the 5 percent level. After differencing to convert the data to growth rates, the KPSS test fails to reject stationarity, and the Dickey-Fuller test rejects the presence of a second unit root at the 5 and even at the 1 percent level. We conclude that industrial production has one unit root, though there is a possibility of no unit roots.

The tests on levels of producer prices are consistent in that both imply lack of stationarity. The KPSS test fails to reject stationarity for the growth rate, but the Dickey-Fuller test fails to reject the presence of a second unit root at the 5 percent level, though it does reject at the 10 percent level. Accordingly, we conclude that producer prices have a single unit root, though there is a possibility of two unit roots.[3]

We also performed Engle-Granger tests for co-integration. Tests on each possible pair of variables failed to reject the null hypothesis of non-co-integration, leading us to conclude that the three series contain at least two unit roots. However, tests of the three variables together showed signs that the group is co-integrated, implying there are at most two unit roots. When the test was performed with producer prices as the dependent variable in the first-stage regression, the null hypothesis of non-co-integration failed to be rejected, but when it was performed with industrial production as the dependent variable the null hypothesis was rejected at the 10 percent level, and when it was performed with the interest rate as the dependent variable, non-co-integration was rejected at the 5 percent level.

We therefore concluded that it was reasonable to construct a model of output, prices, and the interest rate

3. The null hypothesis that producer prices have three unit roots was strongly rejected by a Dickey-Fuller test.

that contains exactly two unit roots (or independent permanent shocks). We shall assume that one of these permanent shocks is a shock to aggregate supply and the other is a shock to aggregate demand.

3. The economic model

The model we examine is based on Rogoff (1985). Because of its simple structure, it has been used extensively to analyze optimal monetary policy under rational expectations. While such models are susceptible to the *a priori* criticism that the estimated parameters do not reflect a "deep structure" of optimizing behavior, McCallum (1989, 102-107) has argued that any model where the supply function has classical properties is for many purposes similar to models derived from explicit maximizing behavior.

3.1 Aggregate Supply

Following Rogoff (1985) and Barro and Gordon (1983), we assume supply increases (relative to trend) when current prices (adjusted for trend) rise above the rationally expected prices based on the previous period's information. Lucas (1973) provides a justification for such an effect when suppliers are confused about whether shocks are primarily local (and real) or aggregate (and nominal). Our model does not distinguish between local and aggregate shocks, while agents always know the current demand and supply shocks. They may be confused only about the permanence of those shocks. Nevertheless, we can obtain an analog of the Lucas supply curve if we assume suppliers base their expectations on last period's information. Alternatively, Fischer (1977) generates such a supply curve in a model where suppliers pre-commit to contracts one period in advance of producing output. The contracting interpretation is used by Rogoff (1985) to justify a positive supply response to price increases above last period's expected level.

In a departure from Rogoff (1985), we allow supply to be autocorrelated. This could result, for example, from investments that transmit current deviations of supply into future periods. Thus, the aggregate supply curve can be written (where all variables are in logarithms):

$$y_t = \rho y_{t-1} + \gamma(p_t - E_{t-1}p_t) + s_t \qquad (1)$$

where the supply shock s_t at t is a *combined* temporary and permanent shock. Specifically, we assume

$$s_t - s_{t-1} = s_t^P + s_t^T - s_{t-1}^T \qquad (2)$$

where the shocks s^P (the innovation to the permanent component of the overall supply shock) and s^T (the temporary component) are assumed to be uncorrelated at all leads and lags and each of them is assumed to be independently identically distributed (*iid*). Because we use GMM for estimation, we do not need to specify a distribution for the shocks s^P and s^T. We merely need to assume that both shocks have a zero mean (since we are measuring deviations of y, p and i about linear time trends) and finite second moments. The same is true of the components of the demand shocks that are specified below.

Temporary supply shocks could represent the effect of strikes, severe weather or other temporary influences on aggregate production. Permanent supply shocks represent long-lasting shifts in aggregate supply associated, for example, with changes in technology and factor supplies.[4]

While s_t is known at t, we assume the observing econometrician can never recover a time path for the permanent and temporary components and is thus restricted to estimating the contributions of the different types of shocks to variances and covariances. We consider several models in so far as agents are concerned. We allow agents to know the permanence of shocks in the same period they occur, only after one period, or only after two periods.[5]

We shall assume that the number of integrated random variables among the driving shocks matches the number of non-stationary driving shocks indicated by the unit-root and co-integration analysis. The structural model then must be constructed so that it would yield stationary endogenous variables if the driving shocks had also been stationary. In particular, the autocorrelation parameter, ρ needs to lie in the interval (-1,1). We expect the elasticity coefficient γ to be positive.

3.2 Aggregate Demand

Aggregate demand is assumed to reflect intertemporal substitution and, if substitution effects dominate, to respond negatively to the current real interest rate. Moreover, if the lagged real interest rate shifts demand toward the present from the previous period, the lagged real interest rate could positively affect current aggregate demand.[6] As in Rogoff (1985) and Sieper (1989), we assume that, in contrast to factor markets where expectations are based on information available at t-1, expectations in capital markets are based on information available at t. We also allow aggregate demand to be autocorrelated.

To begin with, we examine data on only output growth and inflation, which restricts our ability to identify shocks and parameters. We use the results of this investigation, however, to expand the model in a number of ways. In particular, we later allow for separate real and nominal aggregate demand shocks, while we also allow monetary policy to react to the real demand and supply shocks. The availability of interest rate

4. Brunner, Cukierman and Meltzer (1980) developed a similar theoretical model with both permanent and temporary shocks.

5. We implicitly assume that agents know more about economic shocks than the price, output and interest rate data we use.

6. In another specification, we followed McCallum and Nelson (1996) who argue that dynamic optimization implies that aggregate demand also depends positively on $E_t y_{t+1}$. Empirically, it proved difficult to distinguish the effects of $E_t y_{t+1}$ and $E_t p_{t+1}$ on aggregate demand. We therefore focused on the simpler case where $E_t y_{t+1}$ is omitted.

data also allows us to test more complicated demand or supply curves. Thus, the aggregate demand curve in the general case can be written (with variables other than the interest rate in logarithms):

$$y_t = \eta y_{t-1} - \alpha(i_t - E_t p_{t+1} + p_t) + \lambda(i_{t-1} - E_{t-1} p_t + p_{t-1}) + r_t \tag{3}$$

where we expect $\alpha, \lambda > 0$ and[7] $1 > \eta > -1$. The real shock r_t represents shifts in aggregate demand such as changes in demographics, fiscal policy or export demand.

3.3 Money Market

We also postulate a conventional aggregate demand for money balances (relative to trend):

$$m_t - p_t = \beta y_t - \delta^{-1} i_t + m_t^D \tag{4}$$

where m_t^D is a shock to money demand and δ^{-1} is the interest semi-elasticity of the demand for money.

3.4 Reduced form aggregate demand curve

We assume equilibrium p_t and i_t equate aggregate supply and aggregate demand for goods and money. From the money market equilibrium condition and (4) we can conclude that

$$i_t = \beta\delta y_t - \delta(m_t^S - m_t^D - p_t) \tag{5}$$

where m_t^S is a shock to money supply (relative to trend). Substitute (5) into the aggregate demand curve (3) to deduce that it can be written:

$$\begin{aligned}(1+\alpha\beta\delta)y_t = (\eta+\lambda\beta\delta)y_{t-1} - \alpha(1+\delta)p_t + \lambda(1+\delta)p_{t-1} + \alpha E_t p_{t+1} - \lambda E_{t-1} p_t + \\ \alpha\delta(m_t^S - m_t^D) - \lambda\delta(m_{t-1}^S - m_{t-1}^D) + r_t\end{aligned} \tag{6}$$

When $\lambda = 0$, the reduced form aggregate demand curve (6) has a composite error term

$$d_t = \frac{\alpha\delta(m_t^S - m_t^D) + r_t}{1 + \alpha\beta\delta} \tag{7}$$

and initially we shall assume neither the public nor the econometrician observe m_t^S, m_t^D or r_t. Nevertheless, using current and lagged y, p, $E_t p_{t+1}$ and (6) the public can infer the value of the amalgamated demand shock d. When $\lambda = 0$, we can write the aggregate demand curve in terms of prices, the demand shock d and the anticipated supply shock in the form:

$$y_t = \psi y_{t-1} + \Phi E_t p_{t+1} - (\Phi + \Gamma) p_t + d_t \tag{8}$$

where $\psi = \eta/(1+\alpha\beta\delta)$, $\Phi = \alpha/(1+\alpha\beta\delta)$ and $\Gamma = \alpha\delta/(1+\alpha\beta\delta)$ and we shall impose the condition $1 > \psi > -1$

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7. As we shall see below, while we expect to find $1 > \eta > -1$ this condition is not necessary to guarantee stability.

in order to guarantee invertibility of the moving average component in the solutions for Δp_t and Δy_t.

Until we consider data on interest rates, we use (8) for the aggregate demand curve. Analogously to the supply shock s_t we assume that the demand shock d_t is a combined temporary and permanent shock with:

$$d_t - d_{t-1} = d_t^P + d_t^T - d_{t-1}^T \tag{9}$$

The shocks d_t^P and d_t^T are assumed to be *iid* and uncorrelated at all leads and lags with each other and with the supply shocks.

We again assume that agents could know the permanence of shocks in the period they occur, only after one period or only after two periods. As with supply shocks, we also assume that the econometrician can never recover a time path for the permanent and temporary components d_t^P and d_t^T.

3.5 Equilibrium

Using the lag operator L, the aggregate supply curve (1) can be written:

$$(1-\rho L)y_t = \gamma(p_t - E_{t-1}p_t) + s_t \tag{10}$$

while the aggregate demand curve (8) can be written

$$(1-\psi L)y_t = \Phi E_t p_{t+1} - (\Phi + \Gamma)p_t + d_t \ . \tag{11}$$

Multiplying (10) by $(1–\psi L)$ and (11) by $(1–\rho L)$ we deduce that product market equilibrium requires

$$\begin{aligned}(1-\psi L)[\gamma(p_t - E_{t-1}p_t) + s_t] &= (1-\rho L)[\Phi E_t p_{t+1} - (\Phi + \Gamma)p_t + d_t] \\ &= \Phi E_t p_{t+1} - \Phi\rho E_{t-1}p_t - (\Phi + \Gamma)(1-\rho L)p_t + (1-\rho L)d_t\end{aligned} \tag{12}$$

Since the composite shocks s_t and d_t are non-stationary, p_t is also non-stationary. To solve for the equilibrium price and output, we need to manipulate equation (12) to ensure we are working in spaces of stationary processes. By adding and subtracting $\Phi\rho p_t$, equation (12) can be re-arranged to obtain

$$\Phi E_t p_{t+1} - (\Phi + \Gamma)(1-\rho L)p_t - \Phi\rho p_t = (1-\psi L)s_t - (1-\rho L)d_t + (\gamma - \Phi\rho - \psi\gamma L)(p_t - E_{t-1}p_t)\ . \tag{13}$$

Now observe that, if we define the inflation rate $P_t = (1\text{-}L)p_t$, then $p_t – E_{t-1}p_t = P_t – E_{t-1}P_t$ is stationary[8] while

$$(1-L)E_t p_{t+1} = E_t p_{t+1} - E_{t-1}p_t = E_t p_{t+1} - p_t + p_t - E_{t-1}p_t = E_t[(1-L)p_{t+1}] + (p_t - E_{t-1}p_t)\ .$$

Thus, differencing (13), we obtain a stochastic difference equation for P:

$$\begin{aligned}\Phi E_t P_{t+1} - (\Phi + \Gamma)(1-\rho L)P_t - \Phi\rho P_t &= (1-\psi L)(s_t^P + s_t^T - s_{t-1}^T) - (1-\rho L)(d_t^P + d_t^T - d_{t-1}^T) \\ &\quad + [(\gamma - \Phi\rho - \Phi) - (\psi\gamma + \gamma - \Phi\rho)L + \psi\gamma L^2](P_t - E_{t-1}P_t)\end{aligned} \tag{14}$$

8. Thus, while p_t and $E_{t-1}p_t$ are both non-stationary, they are co-integrated.

3.6 Information processing

Individuals know the functional forms of the aggregate demand and supply curves. They also know p_t and y_t, and therefore the values of s_t and d_t, at time t. We assume to begin with, however, that they do not know the decomposition of s_t or d_t into their components s_t^P, s_t^T, d_t^P or d_t^T until period t+1.[9] Based on the form of (14), we deduce that P_t will be a linear function of current and lagged s_t^P, s_t^T, d_t^P and d_t^T. Since individuals know, at t-1, all shocks dated t-2 or earlier, (P_t–$E_{t-1}P_t$) will be a linear sum:[10]

$$P_t - E_{t-1}P_t = \pi_{10}s_t^P + \pi_{20}s_t^T + \pi_{30}d_t^P + \pi_{40}d_t^T + \pi_{11}(s_{t-1}^P - E_{t-1}s_{t-1}^P) + \pi_{21}(s_{t-1}^T - E_{t-1}s_{t-1}^T) + \pi_{31}(d_{t-1}^P - E_{t-1}d_{t-1}^P) + \pi_{41}(d_{t-1}^T - E_{t-1}d_{t-1}^T) \qquad (15)$$

Since individuals know $s_{t-2}^P, s_{t-2}^T, d_{t-2}^P, d_{t-2}^T$, $\Delta s_{t\text{-}1} = s_{t-1}^P + s_{t-1}^T - s_{t-2}^T$ and $\Delta d_{t\text{-}1} = d_{t-1}^P + d_{t-1}^T - d_{t-2}^T$ at t–1 they will also observe $s_{t-1}^P + s_{t-1}^T$ and $d_{t-1}^P + d_{t-1}^T$. Projecting onto these variables they would obtain:

$$E_{t-1}\begin{bmatrix} s_{t-1}^P \\ s_{t-1}^T \end{bmatrix} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}(s_{t-1}^P + s_{t-1}^T) \text{ and } E_{t-1}\begin{bmatrix} d_{t-1}^P \\ d_{t-1}^T \end{bmatrix} = \begin{bmatrix} \mathrm{b}_1 \\ b_2 \end{bmatrix}(d_{t-1}^P + d_{t-1}^T) \qquad (16)$$

where

$$a_1 = \frac{\sigma_{s^P}^2}{\sigma_{s^P}^2 + \sigma_{s^T}^2},\ a_2 = \frac{\sigma_{s^T}^2}{\sigma_{s^P}^2 + \sigma_{s^T}^2} = 1 - a_1,\ b_1 = \frac{\sigma_{d^P}^2}{\sigma_{d^P}^2 + \sigma_{d^T}^2} \text{ and } b_2 = \frac{\sigma_{d^T}^2}{\sigma_{d^P}^2 + \sigma_{d^T}^2} = 1 - b_1 .$$

3.7 ARIMA representations for p_t and y_t

We define the inverse of the lag operator by

$$L^{-1}x_{t-i} = \begin{cases} x_{t-i+1} & i > 0 \\ E_t x_{t-i+1} & i \le 0 \end{cases} \qquad (17)$$

where x_t is known at time t. Then we can show:

Lemma 1: The equilibrium inflation rate P_t satisfies the stochastic difference equation:

$$(\Phi + \Gamma)(1 - FL^{-1})(1 - \rho L)P_t = (1 - \rho L)(d_t^P + d_t^T - d_{t-1}^T) - (1 - \psi L)(s_t^P + s_t^T - s_{t-1}^T) - [\theta_0 - \theta_1 L + \theta_2 L^2]\left[\sum_{i=0}^{1}(\kappa_{1i}s_{t-i}^P + \kappa_{2i}s_{t-i}^T + \kappa_{3i}d_{t-i}^P + \kappa_{4i}d_{t-i}^T)\right] \qquad (18)$$

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9. The simpler case when there is no information lag is discussed in footnotes while the next section discusses the more complicated case where there is a two-period information lag.

10. If there is no information lag, (15) becomes $P_t - E_{t-1}P_t = \pi_{10}s_t^P + \pi_{20}s_t^T + \pi_{30}d_t^P + \pi_{40}d_t^T$.

for constants $F = 1/(1+\delta)$, $\theta_0 = \gamma - \Phi(1+\rho)$, $\theta_1 = \gamma(1+\psi) - \Phi\rho$, $\theta_2 = \psi\gamma$, $\kappa_{i0} = \pi_{i0}$, $i = 1,\ldots,4$, and[11]

$$\kappa_{11} = (\pi_{11} - \pi_{21})a_2\,,\ \kappa_{21} = -(\pi_{11} - \pi_{21})a_1\,,\ \kappa_{31} = (\pi_{31} - \pi_{41})b_2 \text{ and } \kappa_{41} = -(\pi_{31} - \pi_{41})b_1 \tag{19}$$

Proof. The left side of (14) can be written

$$\begin{aligned} \Phi E_t P_{t+1} - (\Phi + \Gamma)(1 - \rho L)P_t - \Phi\rho P_t &= -(\Phi + \Gamma)\left[-\frac{1}{1+\delta}E_t P_{t+1} + \left(1 + \frac{1}{1+\delta}\rho\right)P_t - \rho P_{t-1}\right] \\ &= -(\Phi + \Gamma)\left[\left(1 - \frac{1}{1+\delta}L^{-1}\right)P_t - \left(1 - \frac{1}{1+\delta}L^{-1}\right)\rho P_{t-1}\right] \end{aligned} \tag{20}$$

Also, substitute (16) into (15) and then substitute the result into (14).

Observe that for $\delta > 0$, $F < 1$, while all shocks on the right side of (18) are stationary. The operator in L^{-1} on the left side of (18) can therefore be expanded as a geometric series on the right to show:

Theorem 1: When the composition of shocks is unknown for one period, equilibrium inflation satisfies

$$(1 - \rho L)P_t = \sum_{i=0}^{3}(\pi_{1i}s^P_{t-i} + \pi_{2i}s^T_{t-i} + \pi_{3i}d^P_{t-i} + \pi_{4i}d^T_{t-i}) \tag{21}$$

for constant coefficients π_{ij}, $i = 1,\ldots,4$, $j = 0,\ldots,3$.

Proof. We have, for $i \geq 0$,

$$\frac{1}{1 - FL^{-1}}s^P_{t-i} = s^P_{t-i} + Fs^P_{t-i+1} + \ldots + F^i a_1(s^P_t + s^T_t) \tag{22}$$

$$\frac{1}{1 - FL^{-1}}s^T_{t-i} = s^T_{t-i} + Fs^T_{t-i+1} + \ldots + F^i a_2(s^P_t + s^T_t) \tag{23}$$

with similar expressions for the demand shocks.[12] Inverting $(1-FL^{-1})$ on the right of (18), equating coefficients of shocks, and using $\kappa_{i0} = \pi_{i0}$, $i = 1,\ldots,4$, and expressions (19) for κ_{i1} we find that equilibrium P_t will indeed be given by (21) so long as the π_{ij} coefficients satisfy the following simultaneous equations:

$$(\Phi + \Gamma + \theta_0 - \theta_1 F + \theta_2 F^2)\pi_{10} = (\Phi + \Gamma + \theta_0 - \theta_1 F + \theta_2 F^2)\pi_{20} = -(1 - \psi F)(1 - a_2 F)$$

$$[\Phi + \Gamma + a_2(\theta_0 - \theta_1 F + \theta_2 F^2)]\pi_{11} = \psi + (\theta_1 - \theta_2 F)\pi_{10} + (\theta_0 - \theta_1 F + \theta_2 F^2)\pi_{21}a_2$$

$$[\Phi + \Gamma + a_1(\theta_0 - \theta_1 F + \theta_2 F^2)]\pi_{21} = \psi + (1 - \psi F) + (\theta_1 - \theta_2 F)\pi_{10} + (\theta_0 - \theta_1 F + \theta_2 F^2)\pi_{11}a_1$$

$$(\Phi + \Gamma)\pi_{12} = (\theta_1 - \theta_2 F)(\pi_{11} - \pi_{21})a_2 - \theta_2\pi_{10}$$

$$(\Phi + \Gamma)\pi_{22} = -\psi - (\theta_1 - \theta_2 F)(\pi_{11} - \pi_{21})a_1 - \theta_2\pi_{10}$$

11. When there is no information lag the κ_{i1} coefficients are all zero.

12. When there is no information lag, the final term in (22) simply becomes $F^i s^P_t$ while the final term in (23) becomes $F^i s^T_t$ with analogous modifications for the demand shocks. The moving average in (21) also becomes second order.

$$(\Phi+\Gamma)\pi_{13} = -\theta_2(\pi_{11}-\pi_{21})a_2$$

$$(\Phi+\Gamma)\pi_{23} = \theta_2(\pi_{11}-\pi_{21})a_1$$

$$(\Phi+\Gamma+\theta_0-\theta_1 F+\theta_2 F^2)\pi_{30} = (\Phi+\Gamma+\theta_0-\theta_1 F+\theta_2 F^2)\pi_{40} = (1-\rho F)(1-b_2 F)$$

$$[\Phi+\Gamma+b_2(\theta_0-\theta_1 F+\theta_2 F^2)]\pi_{31} = -\rho+(\theta_1-\theta_2 F)\pi_{30}+(\theta_0-\theta_1 F+\theta_2 F^2)\pi_{41}b_2$$

$$[\Phi+\Gamma+b_1(\theta_0-\theta_1 F+\theta_2 F^2)]\pi_{41} = -\rho-(1-\rho F)+(\theta_1-\theta_2 F)\pi_{30}+(\theta_0-\theta_1 F+\theta_2 F^2)\pi_{31}b_1$$

$$(\Phi+\Gamma)\pi_{32} = (\theta_1-\theta_2 F)(\pi_{31}-\pi_{41})b_2-\theta_2\pi_{30}$$

$$(\Phi+\Gamma)\pi_{42} = \rho-(\theta_1-\theta_2 F)(\pi_{31}-\pi_{41})b_1-\theta_2\pi_{30}$$

$$(\Phi+\Gamma)\pi_{33} = -\theta_2(\pi_{31}-\pi_{41})b_2$$

$$(\Phi+\Gamma)\pi_{43} = \theta_2(\pi_{31}-\pi_{41})b_1$$

Comment: Note that the solution (21) is consistent with the unanticipated inflation rate given in (15).

Use Π_1 for the 4×4 matrix of MA coefficients with Π_{1j} the *j*th column of Π_1, so the 4 polynomials multiplying $z_t' = \left[s_t^P\ s_t^T\ d_t^P\ d_t^T\right]$ are the rows of

$$\Pi_1(L) = \sum_{j=1}^{4} \Pi_{1j}' L^{j-1}$$

Then we can write the ARMA(1,3) representation for P_t as:

$$(1-\rho L)P_t = \Pi_1(L)z_t. \tag{24}$$

From the supply curve (1), (15) and (16) we obtain an expression for equilibrium output:

Theorem 2: When the composition of shocks is unknown for one period, equilibrium output y_t satisfies:[13]

$$(1-\rho L)y_t = s_t+\gamma\sum_{i=0}^{1}(\kappa_{1i}s_{t-i}^P+\kappa_{2i}s_{t-i}^T+\kappa_{3i}d_{t-i}^P+\kappa_{4i}d_{t-i}^T) \tag{25}$$

where $\kappa_{i0} = \pi_{i0}$, $i = 1,\ldots 4$ while κ_{i1}, $i = 1,\ldots 4$, satisfy (19).

Proof. Substitute (16) and the right hand side of (15) into the aggregate supply curve (1).

Corollary: The first difference of the equilibrium output $Y_t = \Delta y_t$ follows an ARMA(1,2) process.[14]

Proof. Multiply (25) through by (1–*L*).

13. When there is no information lag, the moving average in (25) becomes first order.

14. Since output *growth* is stationary, shocks cannot permanently affect it. The long run effect of a shock on the *level* of output can, however, be non-zero. Buiter (1995, note 13) has argued that the restriction, used by Blanchard and Quah (1989) and others, that demand shocks have no long-run real effects, makes sense for nominal, but not real, demand shocks.

If we define a 4×3 matrix Π_2 of MA coefficients, we can write the ARMA(1,2) representation for Y_t:

$$(1-\rho L)Y_t = \Pi_2(L)z_t . \tag{26}$$

As shown in Hartley and Whitt (1997), (24) and (26) can then be used to derive theoretical expressions for the variances and autocovariances of P_t or Y_t and the cross covariances between lags of P_t and Y_t.

3.8 Two periods of uncertainty

If agents do not know the decomposition of s_t or d_t into their components until period t+2 it is possible to show that equilibrium inflation P_t follows an ARMA(1,4) process:

$$(1-\rho L)P_t = \sum_{i=0}^{4}(\pi_{1i}s^P_{t-i} + \pi_{2i}s^T_{t-i} + \pi_{3i}d^P_{t-i} + \pi_{4i}d^T_{t-i}) \tag{27}$$

Specifically, in place of (15), $(P_t - E_{t-1}P_t)$ will now be a linear sum involving two lags:

$$\begin{aligned} P_t - E_{t-1}P_t &= \pi_{10}s^P_t + \pi_{20}s^T_t + \pi_{30}d^P_t + \pi_{40}d^T_t + \pi_{11}(s^P_{t-1} - E_{t-1}s^P_{t-1}) + \pi_{21}(s^T_{t-1} - E_{t-1}s^T_{t-1}) \\ &+ \pi_{31}(d^P_{t-1} - E_{t-1}d^P_{t-1}) + \pi_{41}(d^T_{t-1} - E_{t-1}d^T_{t-1}) + \pi_{12}(s^P_{t-2} - E_{t-1}s^P_{t-2}) + \pi_{22}(s^T_{t-2} - E_{t-1}s^T_{t-2}) \\ &+ \pi_{32}(d^P_{t-2} - E_{t-1}d^P_{t-2}) + \pi_{42}(d^T_{t-2} - E_{t-1}d^T_{t-2}) \equiv \sum_{i=0}^{2}(\kappa_{1i}s^P_{t-i} + \kappa_{2i}s^T_{t-i} + \kappa_{3i}d^P_{t-i} + \kappa_{4i}d^T_{t-i}) \end{aligned} \tag{28}$$

Since at t-1 individuals now know $s^P_{t-2} + s^T_{t-2} - s^T_{t-3}$, $s^P_{t-1} + s^T_{t-1} - s^T_{t-2}$, $d^P_{t-2} + d^T_{t-2} - d^T_{t-3}$, $d^P_{t-1} + d^T_{t-1} - d^T_{t-2}$ and all variables dated t-3 and earlier they effectively observe $s^P_{t-2} + s^T_{t-2}$, $s^P_{t-1} + s^T_{t-1} - s^T_{t-2}$, $d^P_{t-2} + d^T_{t-2}$ and $d^P_{t-1} + d^T_{t-1} - d^T_{t-2}$. Projecting onto these variables they would obtain:

$$E_{t-1}\begin{bmatrix} s^P_{t-1} \\ s^T_{t-1} \\ s^P_{t-2} \\ s^T_{t-2} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{bmatrix}\begin{bmatrix} s^P_{t-1} + s^T_{t-1} - s^T_{t-2} \\ s^P_{t-2} + s^T_{t-2} \end{bmatrix} \text{ and } E_{t-1}\begin{bmatrix} d^P_{t-1} \\ d^T_{t-1} \\ d^P_{t-2} \\ d^T_{t-2} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \\ b_{41} & b_{42} \end{bmatrix}\begin{bmatrix} d^P_{t-1} + d^T_{t-1} - d^T_{t-2} \\ d^P_{t-2} + d^T_{t-2} \end{bmatrix} \tag{29}$$

where the coefficients a_{ij} and b_{ij} (with $a_{31}+a_{41} = 0$, $b_{31}+b_{41} = 0$, $a_{32}+a_{42} = 1$ and $b_{32}+b_{42} = 1$) satisfy

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{bmatrix}\begin{bmatrix} \sigma^2_{s^P} + 2\sigma^2_{s^T} & -\sigma^2_{s^T} \\ -\sigma^2_{s^T} & \sigma^2_{s^P} + \sigma^2_{s^T} \end{bmatrix} = \begin{bmatrix} \sigma^2_{s^P} & 0 \\ \sigma^2_{s^T} & 0 \\ 0 & \sigma^2_{s^P} \\ -\sigma^2_{s^T} & \sigma^2_{s^T} \end{bmatrix} \text{ and } \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \\ b_{41} & b_{42} \end{bmatrix}\begin{bmatrix} \sigma^2_{d^P} + 2\sigma^2_{d^T} & -\sigma^2_{d^T} \\ -\sigma^2_{d^T} & \sigma^2_{d^P} + \sigma^2_{d^T} \end{bmatrix} = \begin{bmatrix} \sigma^2_{d^P} & 0 \\ \sigma^2_{d^T} & 0 \\ 0 & \sigma^2_{d^P} \\ -\sigma^2_{d^T} & \sigma^2_{d^T} \end{bmatrix} \tag{30}$$

Substituting (29) into (28) we also deduce that κ_{ji} and π_{ji}, i = 1,2 are related by the equations:

$$\begin{bmatrix}\kappa_{11}\\ \kappa_{21}\\ \kappa_{12}\\ \kappa_{22}\end{bmatrix} = \begin{bmatrix} 1-a_{11} & -a_{21} & -a_{31} & -a_{41} \\ -a_{11} & 1-a_{21} & -a_{31} & -a_{41} \\ -a_{12} & -a_{22} & 1-a_{32} & -a_{42} \\ a_{11}-a_{12} & a_{21}-a_{22} & a_{31}-a_{32} & 1+a_{41}-a_{42}\end{bmatrix}\begin{bmatrix}\pi_{11}\\ \pi_{21}\\ \pi_{12}\\ \pi_{22}\end{bmatrix} \tag{31}$$

and

$$\begin{bmatrix}\kappa_{31}\\ \kappa_{41}\\ \kappa_{32}\\ \kappa_{42}\end{bmatrix} = \begin{bmatrix} 1-b_{11} & -b_{21} & -b_{31} & -b_{41} \\ -b_{11} & 1-b_{21} & -b_{31} & -b_{41} \\ -b_{12} & -b_{22} & 1-b_{32} & -b_{42} \\ b_{11}-b_{12} & b_{21}-b_{22} & b_{31}-b_{32} & 1+b_{41}-b_{42}\end{bmatrix}\begin{bmatrix}\pi_{31}\\ \pi_{41}\\ \pi_{32}\\ \pi_{42}\end{bmatrix}. \tag{32}$$

Equations (29) updated one period imply, for $i \geq 2$,

$$\frac{1}{1-FL^{-1}}s^P_{t-i} = s^P_{t-i} + Fs^P_{t-i+1} + \ldots + F^{i-2}s^P_{t-2} + F^{i-1}\begin{bmatrix}a_{31}+a_{11}F & a_{32}+a_{12}F\end{bmatrix}\begin{bmatrix}s^P_{t-1}+s^T_{t-1}-s^T_{t-2}\\ s^P_{t-2}+s^T_{t-2}\end{bmatrix}$$

$$\frac{1}{1-FL^{-1}}s^T_{t-i} = s^T_{t-i} + Fs^T_{t-i+1} + \ldots + F^{i-2}s^T_{t-2} + F^{i-1}\begin{bmatrix}a_{41}+a_{21}F & a_{42}+a_{22}F\end{bmatrix}\begin{bmatrix}s^P_{t-1}+s^T_{t-1}-s^T_{t-2}\\ s^P_{t-2}+s^T_{t-2}\end{bmatrix}$$

while

$$\frac{1}{1-FL^{-1}}\begin{bmatrix}s^P_{t-1}\\ s^T_{t-1}\end{bmatrix} = \begin{bmatrix}a_{31}+a_{11}F & a_{32}+a_{12}F\\ a_{41}+a_{21}F & a_{42}+a_{22}F\end{bmatrix}\begin{bmatrix}s^P_{t-1}+s^T_{t-1}-s^T_{t-2}\\ s^P_{t-2}+s^T_{t-2}\end{bmatrix}$$

$$\frac{1}{1-FL^{-1}}\begin{bmatrix}s^P_{t}\\ s^T_{t}\end{bmatrix} = \begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}\begin{bmatrix}s^P_{t-1}+s^T_{t-1}-s^T_{t-2}\\ s^P_{t-2}+s^T_{t-2}\end{bmatrix}$$

Also note that, since the combination of shocks $s^P_t + s^T_t - s^T_{t-1}$ is known at t:

$$\frac{1}{1-FL^{-1}}(s^P_t + s^T_t - s^T_{t-1}) = s^P_t + s^T_t - s^T_{t-1} - F[a_{21}(s^P_t + s^T_t - s^T_{t-1}) + a_{22}(s^P_{t-1} + s^T_{t-1})]$$

$$\frac{1}{1-FL^{-1}}(s^P_{t-1} + s^T_{t-1} - s^T_{t-2}) = s^P_{t-1} + s^T_{t-1} - s^T_{t-2} + F[(1-a_{21}F)(s^P_t + s^T_t - s^T_{t-1}) - a_{22}F(s^P_{t-1} + s^T_{t-1})]$$

$$\begin{aligned}\frac{1}{1-FL^{-1}}(s^P_{t-2} + s^T_{t-2} - s^T_{t-3}) &= s^P_{t-2} + s^T_{t-2} - s^T_{t-3} + F(s^P_{t-1} + s^T_{t-1} - s^T_{t-2})\\ &+ F^2[(1-a_{21}F)(s^P_t + s^T_t - s^T_{t-1}) - a_{22}F(s^P_{t-1} + s^T_{t-1})]\end{aligned}$$

Analogous expressions can be derived for the demand shocks. The operator $(1-FL^{-1})$ can again be inverted on the right side of (14), allowing us to deduce that equilibrium P_t will indeed be given by (27).

Equilibrium output can also be shown to satisfy:

$$(1-\rho L)y_t = s_t + \gamma \sum_{i=0}^{2} (\kappa_{1i} s^P_{t-i} + \kappa_{2i} s^T_{t-i} + \kappa_{3i} d^P_{t-i} + \kappa_{4i} d^T_{t-i}) \tag{33}$$

where $\kappa_{i0} = \pi_{i0}$, i = 1,…4 while κ_{ij}, i = 1,…4, j = 1,2 satisfy (31) and (32). Multiplying (33) through by (1–L), we conclude that, under the modified information assumptions, the first difference of equilibrium output $Y_t = \Delta y_t$ follows an ARMA(1,3) process.

4. Estimating the parameters using GMM

We examined lags up to six quarters for the autocovariances and cross covariances. We expected that this would cover a substantial part of typical cyclical fluctuations while leaving us a reasonable sample size (from the original 151 quarters running from 1960:2 to 1997:4). We thus obtained theoretical expressions for 2 variances and 25 covariances of rates of change of equilibrium output and price. There are 9 parameters in these expressions. We can write the vector of parameters to be estimated as[15]

$$b = [\rho, \psi, \gamma, \Gamma, F, \sigma_{s^P}, \sigma_{s^T}, \sigma_{d^P}, \sigma_{d^T}] \tag{34}$$

and we can denote the 27×1 vector of theoretical second moments by $\theta(b)$.

From the data, we have N observations on trend-corrected and seasonally adjusted quarterly rates of change in industrial production and producer prices. Using this data, we calculate 27×N cross products corresponding to our 27 theoretical second moments, with one set of cross products for each period n. Following the notation of Hansen (1982), we write $f(\Delta x_n, b)$ for the 27×1 vector of differences between the sample cross products in period n and the corresponding theoretical second moments in $\theta(b)$. Under the null hypothesis, $E[f(\Delta x_n, b)] = 0$. We form

$$g_N(b) = \frac{1}{N} \sum_{n=1}^{N} f(\Delta x_n, b),$$

which, in our case, equals the vector of differences between the empirical second moments and the corresponding theoretical second moments.

Initial estimates $\hat{b}$ of b are obtained by minimizing the sum of squared errors $g_N(b)' g_N(b)$.[16] Following

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15. We estimated standard deviations to impose the restriction that the variances are non-negative. Also, we can estimate any two parameters from Γ, Φ, δ and F and impose the restrictions implied by the relationships $F = 1/(1+\delta) = \Phi/(\Phi+\Gamma)$. We estimated the inverse hyperbolic tangents of F, ρ and ψ to impose the conditions $|F| < 1$, $|\rho| < 1$ and $|\psi| < 1$.

Hansen (1982), Cumby, Huizinga and Obstfeld (1983) and White and Domowitz (1984) we conclude that $\sqrt{N}(\hat{b}-b)$ will converge in distribution to a random vector with mean zero and covariance matrix

$$(D'D)^{-1}D'SD(D'D)^{-1}$$

where

$$D = E\left[\frac{\partial}{\partial b}f(b)\right]$$

and the matrix S is defined by

$$S = \sum_{j=-\infty}^{\infty} E[f(\Delta x_0, b)f(\Delta x_{-j}, b)'] .$$

An estimate of D can be obtained using the least square parameter estimates $\hat{b}$:

$$\hat{D} = \left[\frac{\partial}{\partial b}g_N(\hat{b})\right] = -\left[\frac{\partial}{\partial b}\theta(\hat{b})\right]$$

Following Newey and West (1987) we estimate S by[17]

$$\hat{S}_J = \hat{\Omega}_0 + \sum_{j=1}^{J} w(j,J)[\hat{\Omega}_j + \hat{\Omega}_j'] \tag{35}$$

where $w(j,J) = 1-[j/(J+1)]$ is a linearly declining weighting function and

$$\hat{\Omega}_j = \frac{1}{N}\sum_{n=j+1}^{N} f(\Delta x_n, b)f(\Delta x_{n-j}, b)' . \tag{36}$$

Hansen (1982) shows that the optimal GMM estimator (in the sense that the asymptotic covariance matrix of b is as small as possible) is obtained by minimizing a weighted sum of squares[18] $g_N(b)'Wg_N(b)$, for a symmetric weighting matrix W which is a consistent estimator of S^{-1}. If we let $\tilde{b}$ be the parameter vector which minimizes this weighted sum of squares then $\sqrt{N}(\tilde{b}-b)$ will converge in distribution to a random vector with mean zero and covariance matrix $(D'SD)^{-1}$, which can be estimated by

$$(\hat{D}'\hat{S}_J\hat{D})^{-1} \tag{37}$$

16. In practice, the numerical minimization algorithm worked better when we normalized by re-scaling parameter values and dividing $g_N(b)'g_N(b)$ by the sum of squared values of the sample moments. We used a combination of a derivative-based quasi-Newton method and the Nelder-Mead simplex algorithm to minimize the highly non-linear objective function. The simplex algorithm proved more effective at finding the general region of parameter space where a minimum lies, while the derivative-based algorithm was more effective at actually attaining the local minimum to be found in that region. To ensure we obtained a global minimum of the objective function, we tried many different starting values for the parameters.

17. In our empirical analysis, we used $J = 12$.

18. In effect, the weighting matrix emphasizes those moments that can be estimated more precisely from the data.

Following the suggestion in Hansen (1982), we test the over-identifying restrictions by evaluating

$$Ng_N(\tilde{b})'(\hat{S}_N)^{-1}g_N(\tilde{b}), \qquad (38)$$

which converges in distribution to a chi-square random variable with k–q degrees of freedom where k is the number of moment conditions and q the number of parameters.

By analogy with variance decompositions in VAR's, we shall use the final parameter estimates to decompose the variances and covariances into the components due to each of the underlying shocks. This will provide our measure of the relative importance of supply and demand, and temporary and permanent shocks in driving output and prices over the sample period.

5. Results using output growth and inflation

The 27 moments used to estimate the model were the variance of output growth, the variance of inflation, each variable's autocovariances up to six quarters, the contemporaneous cross-covariance between output growth and inflation, and other cross-covariances going forward and back up to six quarters. The sample variance of output growth is over double the variance of inflation. The sample variances are graphed alongside the final estimates from the model with a one period information lag in Figure 1.

The pattern of sample cross-covariances graphed in Figure 1 warrants discussion. Kydland and Prescott (1990) and Cooley and Ohanian (1991) report negative cross-covariances between filtered prices and output for the United States at nearly all leads and lags. This led Kydland and Prescott to call the notion of a positive relationship between prices and output a monetary myth.

We find a sizeable negative contemporaneous covariance and a consistently negative cross-covariance between output growth and positive lags of inflation. However, the cross-covariances in the other direction, between output growth and future (negative lags of) inflation, are initially small (relative to the largest cross-covariances in the other direction) and negative, but become small and positive at longer leads.[19]

On the whole, the estimated model matches the sample moments fairly well. For example, the model correctly generates a variance of Y about double that of P, as well as the negative contemporaneous covariance between the two. The most obvious misses occur when a group of sample moments switch signs for

[19]. Differences between our results and those of Cooley and Ohanian and Kydland and Prescott may be the result of either the different measures of output and inflation that we used or the way the data were filtered. Cooley and Ohanian use real GNP and implicit price deflators, while Kydland and Prescott use real GNP and two price measures, the implicit price deflator and consumer price. We use industrial production and producer prices. As for filters, Kydland and Prescott use only the Hodrick-Prescott filter, while Cooley and Ohanian use three filters: linear detrending, differencing, and the Hodrick-Prescott filter. We use differencing but in addition we remove a linear trend and seasonal effects.

different lag lengths. For example, the autocovariance of *Y* is strongly positive at a lag of one quarter, but it turns negative at lags of 5 and 6 quarters. Similarly, the sample cross-covariance between *P* and past values of *Y* is negative when *Y* is lagged 1 to 3 quarters, but turns positive when *Y* is lagged 4 to 6 quarters. The model was unable to reproduce such sign-switching patterns.

The least squares estimates of the parameters, and the corresponding minimized value for the (normalized) sum of squares objective function, are presented in Table 2. We defined the parameters so that all except the autocorrelation coefficients (ρ and ψ) should be positive. If ρ represents lags in the capital accumulation process, however, we would expect it also to be positive. We do require both ρ and ψ to be less than 1 in absolute value. As with ARIMA models, the same autocorrelation structure can be explained either by stationary or non-stationary, and invertible or non-invertible processes. We have eliminated this identification problem by ensuring the numerical algorithm chooses stationary and invertible representations of the data. Similarly, the coefficient *F* on the forward operator is required to be less than 1 in absolute value.

TABLE 2. Least squares parameter estimates

Parameter	1 period information lag	2 period information lag
$\tanh^{-1}(\rho)$	1.1127	1.1127
$\tanh^{-1}(\psi)$	0.1055	0.1055
γ	2.8985	2.9736
Γ	0.8770	0.8770
$\tanh^{-1}(F)$	0.2833	0.2833
σ_{s^P}	0.00516	0.00516
σ_{s^T}	0.01023	0.01253
σ_{d^P}	0.01781	0.01781
σ_{d^T}	3.473e-05	3.389e-06
LS objective	0.12788	0.12788

The sum of squared differences between the sample and theoretical second moments was normalized by dividing by the sum of the squared second moments. The value of the minimized least squares objective function (the last line of Table 2) can therefore be thought of as a type of R^2 measure. Thus, the theoretical model explains about 87% of the “variation” in the sample second moments under consideration.

The least squares objective function is not, however, the best measure of the fit between the theoretical model and the data. The minimized least squares objective function, in common with “calibration” exercises, places greater weight on explaining the larger moments (in absolute value). By contrast, the GMM, or weighted least squares procedure, also emphasizes moments that can be estimated more precisely from the data in the sense that they have a lower sample variance. The weighted least squares estimates, together with their standard errors estimated according to (37), are presented in Table 3.

TABLE 3. Weighted least squares parameter estimates[a]

Parameter	1 period information lag	2 period information lag
$\tanh^{-1}(\rho)$	1.1812[b]	1.1813[b]
	(0.0873)	(0.0884)
$\tanh^{-1}(\psi)$	0.1215[b]	0.1215[b]
	(0.0423)	(0.0434)
γ	2.9043[b]	2.9918[b]
	(0.2589)	(0.5872)
Γ	0.8373[b]	0.8373[b]
	(0.1571)	(0.0917)
$\tanh^{-1}(F)$	0.3126	0.3126
	(0.3042)	(0.3179)
σ_{s^P}	0.004458[b]	0.004458[b]
	(0.00103)	(0.00059)
σ_{s^T}	0.010811[b]	0.012808[b]
	(0.00177)	(0.00426)
σ_{d^P}	0.016556[b]	0.016556[b]
	(0.00226)	(0.00194)
σ_{d^T}	2.141e-07	4.178e-08
	(0.57592)	(39.081)
χ^2 statistic	9.1907	9.1907
(P-value)	(0.955)	(0.955)
Implied parameter values[c]		
ρ	0.8278	0.8279
ψ	0.1209	0.1209
F	0.3028	0.3028
Φ	0.3636	0.3636
δ^{-1}	0.4342	0.4342

a. Standard errors are in parentheses below each parameter estimate.

b. The coefficient is statistically significantly different from zero at conventional levels under the asymptotic t-distribution for the ratio of coefficient to its standard error.

c. The income elasticity of money demand β, the autocorrelation in demand η and the real interest elasticity of demand α cannot be recovered.

The estimated parameters under the different information lags are quite similar, suggesting that the specification of the length of the information lag does not make much difference to the ability of the model to account for the second moments. In the following discussion, we will concentrate on the model with a one period information lag.

All of the parameter estimates have the hypothesized sign and most of them also have “reasonable” magnitudes. The estimates are close to those we previously obtained (Hartley and Whitt, 1997) for the same model using data from the five largest West European economies. The elasticity parameters are perhaps closest to the values we found for the Netherlands, while the standard deviations of shocks are most similar to the values we found for Germany. As with the European economies, supply autocorrelation ρ is estimated with a low standard error and the numerical magnitude suggests substantial persistence.

The inverse of γ can be interpreted as the slope coefficient in an expectations-augmented Phillips curve. The estimated value is similar to that found for all European economies except the U.K. and would appear reasonably consistent with other estimates of similar parameters.

The estimated semi-elasticity of money demand with respect to the nominal interest rate (δ^{-1}) would appear to be below conventional estimates of this parameter. It exceeds our estimates for Germany, the U.K. and Italy but is considerably lower than the values we found for France and the Netherlands.

The estimated standard deviations of the shocks suggest that permanent demand shocks have predominated in U.S. business cycles over this period, followed by temporary and then permanent supply shocks. Temporary demand shocks appear to have been absent.

The relative contributions of the different shocks to variances, autocovariances and cross-covariances in output growth and inflation depend not only on the estimated standard errors of the shocks but also on the autoregressive and moving average coefficients. In the VAR literature, the traditional way to present the information contained in the estimated coefficients is to graph the impulse response functions. Using the parameter estimates in Table 3 we can calculate the effects on Y and P of a unit shock to s_t^P, s_t^T, d_t^P or d_t^T. The resulting impulse response functions for a period of 12 quarters (3 years) when there is only a one quarter information lag are graphed in Figure 2.

Permanent supply shocks have the longest lasting effects on output growth, with the peak positive effects occurring after a two quarter lag. The effects of the remaining shocks on output growth are small beyond one quarter after the period of the shock. Permanent supply shocks also have the longest lasting effects on inflation, although permanent demand shocks also have a persistent positive impact on inflation. The effects of the temporary shocks change sign as we move from the impact to subsequent periods.

TABLE 4. Long run effects of shocks on price and output levels

	1 period information lag		**2 period information lag**	
Shock	***p***	***y***	***p***	***y***
s^P	–6.09869	5.80826	–6.09988	5.80920
s^T	4.066e-05	5.227e-05	2.556e-05	–2.614e-05
d^P	0.99989	–6.864e-10	1.00011	–2.633e-11
d^T	–4.510e-17	–1.162e-04	1.162e-05	1.162e-04

The *cumulative* effects of shocks on output growth and inflation can also be interpreted as long run effects on the output and price *levels*. From the sums of the impulse responses in Figure 2, and using the fact that subsequent coefficients decline exponentially from the final coefficients at lag 12, we can calculate the long run effects of each shock on output and price levels. These are presented for both models in Table 4.

The long run effects of the temporary shocks have to be zero because P and Y are stationary by construction. The corresponding numbers in Table 4 only differ from zero as a result of rounding error in our calculations. Hence, it is also clear that, up to the same rounding error bound, the permanent demand shocks have no effect on the long run level of output and a proportional effect on the price level. This suggests that the permanent demand shocks are predominantly nominal in character.

6. Including interest rates in the analysis

If the interest rate is observable along with y_t and p_t, the value of the monetary shock $m_t^S - m_t^D$ is observable using (5). The public could then observe the monetary and real parts of the shock to aggregate demand, rather than just the amalgamated shock d_t of (7). Following the above results that showed that the permanent component of the demand shock seems to be nominal, we now assume the real demand shock r_t is stationary while the nominal demand shock has both permanent and temporary components. We also allow the monetary authority to react to temporary demand and supply shocks, rather than requiring monetary shocks to be orthogonal to all the other shocks in the system. We reasoned that such an activist policy might be masking some of the effects of temporary shocks in the results in Section 5. Accordingly, we now keep the monetary and real shocks to demand separate and assume that

$$m_t^S - m_t^D = m_t^P + \mu_s(s_t^P + s_t^T) + \mu_d r_t + n_t^T = m_{t-1}^P + n_t^P + \mu_s(s_t^P + s_t^T) + \mu_d r_t + n_t^T \qquad (39)$$

where the coefficients μ_s and μ_d represent the response of monetary policy to supply and real demand shocks while m^P, with *iid* innovation n_t^P, is a permanent shock to excess money supply (relative to a deterministic trend) and n_t^T is a temporary nominal shock. The Federal Reserve is assumed to know r_t and the unexpected innovation in the combined supply shock s_t but not the individual components s^P or s^T. The temporary nominal shock n_t^T should be interpreted as the *residual noise* affecting temporary excess money supply after eliminating the feedback effects due to activist monetary policy.

Adding interest rates while keeping 6 lags adds an additional 33 moments.[20] Observation of the moments involving the change in interest rates also separately identifies β and α. Since we can estimate more parameters, we added the lagged real interest rate to the demand curve. Along with the new parameters specified in (39), the number of parameters to be estimated increases from 9 to 14:

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20. Thus, we now have 6 distinct contemporaneous variances and covariances, 18 autocovariances of output growth, inflation and the change in interest rates, and 36 cross covariances. This is a total of 60 moments compared with the previous 27.

$$b = [\rho, \psi, \gamma, \alpha, \lambda, \delta, \beta, \mu_s, \mu_d, \sigma_{s^P}, \sigma_{s^T}, \sigma_{n^P}, \sigma_{n^T}, \sigma_r] .$$

6.1 Equilibrium prices with lagged real interest rates in the demand curve

When $\lambda \neq 0$, the reduced form aggregate demand curve (6) can be written:

$$y_t = \frac{\eta + \lambda\beta\delta}{1 + \alpha\beta\delta} y_{t-1} - \frac{\alpha(1+\delta)}{1+\alpha\beta\delta} p_t + \frac{\lambda(1+\delta)}{1+\alpha\beta\delta} p_{t-1} + \frac{\alpha}{1+\alpha\beta\delta} E_t p_{t+1} - \frac{\lambda}{1+\alpha\beta\delta} E_{t-1} p_t + \frac{\alpha\delta}{1+\alpha\beta\delta}(m_t^S - m_t^D) - \frac{\lambda\delta}{1+\alpha\beta\delta}(m_{t-1}^S - m_{t-1}^D) + \frac{r_t}{1+\alpha\beta\delta} \quad (40)$$

If we now define $\psi = (\eta+\lambda\beta\delta)/(1+\alpha\beta\delta)$ and proceed as above using (40) in place of (8) we can show that the equilibrium inflation rate now satisfies the difference equation:

$$\alpha(1+\delta)\left(1 - \frac{1}{1+\delta}L^{-1}\right)\left(1 - \frac{\lambda}{\alpha}L\right)(1-\rho L)P_t = -(1+\alpha\beta\delta)(1-\psi L)(s_t^P + s_t^T - s_{t-1}^T) + (1-\rho L)(r_t - r_{t-1}) + \delta(\alpha - \lambda L)(1-\rho L)[n_t^P + \mu_s(s_t^P + s_t^T - s_{t-1}^P - s_{t-1}^T) + \mu_d(r_t - r_{t-1}) + n_t^T - n_{t-1}^T] - (\theta_0 - \theta_1 L + \theta_2 L^2)(P_t - E_{t-1}P_t) \quad (41)$$

where $\theta_0 = \gamma(1+\alpha\beta\delta) - \lambda - \alpha(1+\rho)$, $\theta_1 = \gamma(1+\psi)(1+\alpha\beta\delta) + \lambda(1+\rho) - \alpha\rho$ and $\theta_2 = \psi\gamma(1+\alpha\beta\delta) - \lambda\rho$.

The operator $[1 - FL^{-1}]$, with $F = 1/(1+\delta)$, can again be expanded forward on the right side of (41) to yield an ARMA(2,3) as the solution for equilibrium inflation in the case where the permanence of shocks is unknown for at most one period. If the permanent versus temporary composition of shocks is unknown for two periods equilibrium inflation will follow an ARMA(2,4) process.

From the first difference of aggregate supply (1) we again deduce that equilibrium output growth will be an ARMA(1,2) when the composition of shocks is unknown for one period and an ARMA(1,3) when the composition of shocks is unknown for two periods. The sole AR coefficient in output growth is ρ.

From the money market equilibrium condition (5), the change in the equilibrium interest rate becomes:

$$\left(1 - \frac{\lambda}{\alpha}L\right)(1-\rho L)(i_t - i_{t-1}) = \beta\delta\left(1 - \frac{\lambda}{\alpha}L\right)(1-\rho L)Y_t + \delta\left(1 - \frac{\lambda}{\alpha}L\right)(1-\rho L)P_t - \delta\left(1 - \frac{\lambda}{\alpha}L\right)(1-\rho L)[n_t^P + \mu_s(s_t^P + s_t^T - s_{t-1}^P - s_{t-1}^T) + \mu_d(r_t - r_{t-1}) + n_t^T - n_{t-1}^T] \quad (42)$$

Equation (42) is an ARMA(2, 3) when the permanence of shocks is unknown for at most one period and an ARMA(2,4) when the permanent versus temporary composition of shocks is unknown for two periods.

7. Results when interest rates are included

A representative selection of results is presented in Table 5. As with the model without interest rates, changing the information lag made little difference to the parameter estimates. Estimates were more sensitive, however, to the inclusion of a lagged interest rate in the aggregate demand curve.

The minimized value of the least squares objective was quite close to the value attained for moments involving output growth and inflation alone. Despite a more than doubling of the number of moments to be explained, and an increase from only 9 to 11 parameters for the base model in Table 5, the minimized least squares objective only increased by about 20%. This might be explained by the fact that the moments involving interest rates tend to be much smaller than the moments involving output growth and inflation. However, the model still has to reproduce those small moments, while simultaneously delivering large moments for the other variables, in order to obtain a low minimized sum of squares.

TABLE 5. Least squares parameter estimates

Parameter	Base model		Lagged real interest rate in demand, activist monetary policy	
	No information lag	2 period information lag	No information lag	1 period information lag
$\tanh^{-1}(\rho)$	1.1705	1.1681	1.2077	1.2075
$\tanh^{-1}(\psi)$	0.1720	0.1681	0.4467	0.4463
γ	3.4821	3.4714	3.6491	3.6479
α	4.5852	4.5924	3.5902	3.5929
λ	—	—	0.9299	0.9297
δ^{-1}	1.0262	0.9976	1.0411	1.0406
β	1.1165	1.1141	1.1089	1.1089
μ_s	—	—	–0.0068	–0.0077
μ_d	—	—	0.0213	0.0214
σ_{s^P}	0.00463	0.00465	0.00441	0.00441
σ_{s^T}	5.59e–07	2.31e–07	1.62e–06	8.34e–06
σ_{n^P}	0.01620	0.01627	0.01625	0.01625
σ_{n^T}	0.00154	4.26e–06	6.65e–10	2.41e–07
σ_r	0.00580	0.00568	0.03364	0.03365
LS objective	0.155666	0.155674	0.150965	0.150965

Many of the parameters that are common to Table 2 and Table 5 have similar estimated values. The main exceptions would appear to be δ^{-1} (= $F/(1–F)$) and σ_{s^T} . In particular, temporary supply shocks s^T were estimated to have one of the highest variances in the model with only output growth and inflation but are estimated to be negligible when we add moments involving changes in interest rates. This is so whether or not we allow monetary policy to respond to such shocks. Temporary monetary shocks (or in the case of activist policy the residual shock after allowing for policy feedback) appear to play a minor role in all cases except the base model in Table 5 when there is no information lag.

The weighted least squares parameter estimates are presented in Table 6. The chi-squared statistic measuring the overall fit between the model and the data is not much higher in Table 6 than it was in Table 3 despite the substantial increase in degrees of freedom from 18 to 46 or 49. In fact, the value of the chi-squared statistic in Table 6 is so low that it is doubtful this statistic is truly chi-squared distributed with the hypothesized degrees of freedom in samples as small as ours. Nevertheless, the graph in Figure 3 of

TABLE 6. Weighted least squares parameter estimates[a]

Parameter	Base model		Lagged real interest rate in demand, activist monetary policy	
	No information lag	2 period information lag	No information lag	1 period information lag
$\tanh^{-1}(\rho)$	1.1749[b] (0.0262)	1.1721[b] (0.0267)	1.2191[b] (0.02992)	1.2164[b] (0.02839)
$\tanh^{-1}(\psi)$	0.1733[b] (0.0126)	0.1677[b] (0.0109)	0.4440[b] (0.02501)	0.4481[b] (0.02015)
γ	3.4897[b] (0.1034)	3.4760[b] (0.1066)	3.6079[b] (0.26717)	3.7154[b] (0.31486)
α	4.5795[b] (0.3121)	4.6109[b] (0.2946)	3.6745[b] (0.47121)	3.5686[b] (0.23258)
λ	—	—	0.9469[b] (0.12078)	0.9186[b] (0.06474)
δ^{-1}	1.0359[b] (0.0615)	0.9994[b] (0.0393)	1.0387[b] (0.06470)	1.0463[b] (0.05861)
β	1.1203[b] (0.0231)	1.1163[b] (0.0263)	1.1151[b] (0.02919)	1.1155[b] (0.02615)
μ_s	—	—	–0.00315 (0.13171)	0.03556 (0.13646)
μ_d	—	—	0.02044 (0.01695)	0.02545 (0.01532)
σ_{s^P}	0.00456[b] (0.00020)	0.00460[b] (0.00016)	0.00428[b] (0.00022)	0.00429[b] (0.00023)
σ_{s^T}	5.27e–07 (0.59607)	2.48e–08 (0.75296)	0.00425 (0.00784)	0.00145 (0.00487)
σ_{n^P}	0.01603[b] (0.00036)	0.01613[b] (0.00052)	0.01604[b] (0.00051)	0.01604[b] (0.00052)
σ_{n^T}	0.00163[b] (0.00067)	8.47e–07 (0.68433)	2.60e–08 (18.2240)	2.18e–08 (1.4086)
σ_r	0.00576[b] (0.00028)	0.00561[b] (0.00023)	0.03334[b] (0.00262)	0.03291[b] (0.00195)
χ^2 statistic	11.93620	11.96600	11.65470	11.73430
Implied parameter values				
ρ	0.8258	0.8249	0.8394	0.8386
ψ	0.1715	0.1661	0.4170	0.4203
η	1.0212	1.0218	1.0453	1.0400
F	0.5088	0.4998	0.5095	0.5113
Φ	0.7693	0.7497	0.7431	0.7427
Γ	0.7427	0.7501	0.7154	0.7099

a. Standard errors are in parentheses below each parameter estimate.

b. The coefficient is statistically significantly different from zero at conventional levels under the asymptotic t-distribution for the ratio of coefficient to its standard error.

the 60 sample and estimated moments for the model in the final column of Table 6 also indicates a reasonably good fit between the sample and estimated moments.[21]

Comparing Figures 3 and 1, it seems that adding interest rates has little effect on the fit to the moments included in Figure 1. As before, the model captures major features of the data such as the variances of output growth and inflation, but misses when a group of moments switches sign as the lag length changes.

As for the moments involving the interest rate, the model does well on the variance of $\Delta i_t = i_t - i_{t-1}$ and its contemporaneous covariances with *Y* and *P*, as well as on the positive cross-covariances between *P* and lagged values of Δi. The model correctly matches the positive sign of the cross-covariances between Δi and lagged values of *Y*, but tends to underestimate their size. The autocovariances of Δi and the cross-covariances between *Y* and lagged Δi, as well as those between Δi and lagged *P*, reverse their sign and, as before, the model is unable to match such patterns.

The estimates of autocorrelation in supply are very close in Table 3 and Table 6, with the small range of estimates in Table 6 bracketing the values in Table 3. This parameter is also estimated to have a very small standard error in all cases. The autocorrelation in reduced form aggregate demand ψ is estimated to be higher when interest rates are included. This might be expected in the model with a lagged real interest rate in demand since ψ is then $(\eta+\lambda\beta\delta)/(1+\alpha\beta\delta)$ instead of $\eta/(1+\alpha\beta\delta)$. The estimated interest semi-elasticity of demand for money (δ^{-1}) is close to one throughout Table 6, somewhat larger than the 0.4 implied by the estimate of $F = 1/(1+\delta)$ in Table 3. However, the estimates of *F* in Table 3 are accompanied by a high standard error. It is perhaps not surprising that adding moments involving the interest rate enables us to obtain much more precise estimates of the interest semi-elasticity of money demand.

The results in Table 6 imply η (demand autocorrelation) is greater than one. The overall model is still stable, however, since increases in *y* raise money demand and hence interest rates. Thus, the estimated autocorrelation ψ in *reduced form* aggregate demand is substantially less than 1. A value of η in excess of unity nevertheless implies that, *if real interest rates could be held fixed*, a shock to demand would be unstable. Although the large estimated value of η may reflect a misspecified lag structure elsewhere in the model, adding the lagged real interest rate to the demand curve slightly *raised* the estimated value of η.

On the whole, the estimates in Table 6 of the other elasticities appear reasonable. The estimate of γ, the responsiveness of supply to an unexpected rise in prices (or the inverse of the slope of an expectations augmented Phillips curve), is consistent across all four models in Table 6 and implies that a one-percent

[21]. The other models in Table 6 produced a similar fit.

rise in prices raises output supplied by about $3^1/_2$ percent. In the models without interest rates (Table 3) this parameter was a little smaller, about 3.

Including the interest rate makes it possible to identify the income elasticity of money demand β and the elasticity of aggregate demand with respect to the real interest rate, α. The income elasticity of money demand is consistently estimated as 1.1 throughout Table 6. This value is quite close to many other estimates in the literature. The value of α is estimated to be $4^1/_2$ in the first two models in Table 6, implying that a 1 percentage point rise in today's real interest rate cuts aggregate demand by $4^1/_2$ percent. When the lagged real interest rate and activist monetary policy are added to the model (the last two columns of Table 6), the value of α shrinks somewhat to $3^1/_2$. As for λ, the elasticity of aggregate demand with respect to the lagged real interest rate, its estimate is about 0.9, implying that a one percentage point rise in last quarter's real interest rate (holding today's fixed) *raises* today's aggregate demand by a little less than one percent. The large value of λ relative to its standard error suggests that the lagged real interest rate belongs in the demand curve.

What about the shock terms? The standard deviations of the two permanent shocks, one to supply and one to nominal demand, are consistently estimated and highly significant across all four specifications in Table 6. Moreover, the "typical" permanent demand shock is estimated to be three to four times as large as the "typical" supply shock.

The estimates for the three stationary shocks vary depending on the specification. In the models without activist monetary policy (the first two columns of Table 6), only the real demand shock r_t plays a substantial role; the temporary supply shock is negligible, and the temporary nominal shock is negligible in the model with information lags (column 2) and rather small in the model with no information lag. However, the results in the last two columns suggest that activist monetary policy may be masking some of the effects of temporary shocks.

The parameter μ_s was very poorly estimated. However, μ_d, the reaction of monetary policy to real demand shocks r_t, is consistently positive (albeit with a large standard error) implying that monetary policy accommodates such shocks. Allowing for activist monetary policy results in a much larger estimate of σ_r, making it the largest shock in the model; moreover, σ_r is highly significant. In addition, the standard deviation of the temporary supply shock σ_{s^T} is no longer negligible in size, though it is not significant at conventional levels. On the other hand, the estimated standard deviation of the non-systematic part of the temporary nominal shock σ_{n^T} is completely negligible.

Table 7 presents the fraction of output, inflation, and interest rate variance attributable to each of the five shocks as estimated by the models in the last two columns of Table 6 – the ones with activist monetary policy and the lagged real interest rate in the demand curve. We will focus on the last model, which has a one-period information lag. This model attributes 60 percent of variance in output growth to the permanent nominal shock, with 30 percent attributable to the permanent supply shock. Temporary real demand shocks contribute 8 percent, and temporary supply shocks add 2 percent. The other models also show the permanent nominal shock to be the largest source of output variance.

TABLE 7. Percentage of variance due to each type of shock – lagged real interest rate and activist monetary policy

	No information lag			One period information lag		
	Y	***P***	Δi	***Y***	***P***	Δi
s^P	32.6	63.7	40.4	29.9	62.5	34.7
s^T	0.3	0.8	0.02	2.3	1.7	6.8
n^P	59.6	26.2	3.7	59.9	26.2	3.8
n^T	0.0	0.0	0.0	0.0	0.0	0.0
r	7.4	9.2	55.9	7.9	9.7	54.7

In the real business cycle literature, various papers report that permanent shocks to technology account for over half of U.S. output fluctuations (Kydland and Prescott 1982, 1988, 1991), with Prescott (1986) supporting 75 percent as the best point estimate. In our framework, the permanent (and perhaps temporary) shock to aggregate supply is the best counterpart to technology shocks, but our results indicate a much smaller contribution, about 30 percent. Accordingly, our results undermine the conclusion often reached in the real business cycle literature that U.S. output fluctuations are mainly the result of shocks to technology.

Our model finds a bigger role for supply shocks in explaining inflation variance. About 62 percent of inflation variance is attributed to the permanent supply shock, with 26 percent reflecting the permanent nominal shock. As for the variance of changes in interest rates, the real demand shock is the largest contributor, 55 percent, followed by permanent supply shocks, with 35 percent.

Figure 4 contains impulse response functions for the shocks (other than the negligible temporary nominal shock) in the final model from Table 6. These are analogous to the graphs for the model without interest rates discussed in Section 5. The long run effects of each of the shocks on the *levels* of prices, output and the interest rate are given in Table 8.[22]

Consider first a permanent shock to nominal demand. On impact, a one-percent rise in n^P raises inflation about 0.2 percent, and boosts output growth about 0.7 percent; the rate of interest changes very little. In later quarters the inflation rate remains above its original path, but the impact of the shock gradually dies

22. The estimated long run effects of the shocks were similar in the other models.

away. The cumulative effect on the price level is, to round-off error, unity. As for output growth, after the initial impact the jump in nominal demand has a small negative effect in later quarters, and eventually the level of output returns to its original path.

TABLE 8. Long run effects of shocks on levels of prices, output and the interest rate

Shock	*p*	*y*	*i*
s^P	–6.81264	6.19477	0.09391
s^T	–1.669e-04	–1.776e-15	7.925e-04
n^P	0.99999	–8.428e-12	2.984e-13
n^T	–2.012e-16	3.435e-16	3.337e-05
r	2.920e-05	0.0	–4.588e-05

On impact, a 1 percent positive shock to permanent supply lowers inflation by 0.4 percent. It also lowers output growth about 0.5 percent. This latter effect may seem counter-intuitive, but the model estimates imply that supply is so sensitive to unexpected declines in the price level that the direct positive effect of the shock on supply is initially overwhelmed by the negative effect coming from the decline in p_t. As for the interest rate, the combination of a falling price level and falling output generates a 1 percentage point drop in the interest rate.

In the following quarter, expectations adjust and reduce the difference between p_t and its expected value. The effect on output turns positive, inflation drops even more dramatically, while the interest rate drops slightly more. In subsequent quarters inflation and output growth gradually return to their original rates, but the level of output and interest rates are permanently higher, while the price level is permanently lower. The coefficient of 6.2 on the long run effect of s^P on y implies that a one standard deviation shock (0.0045) produces a long run change in y of about 2.8 percentage points. An upward shift in factor supply or the production function evidently stimulates substantial capital accumulation or productivity improvements elsewhere in the economy so that the multiplier effect on the *level* of y ends up being quite large.

The real demand shock r_t lasts only one period. On impact, a one-percent rise in real demand raises output growth about 0.1 percent, raises inflation slightly, and raises the interest rate about 15 basis points. In the following quarters, the impact effects on output, inflation and the interest rate are gradually reversed.

The autocovariances and cross covariances can be decomposed into components due to each type of shock just as Table 7 decomposed the variances. Figure 5 presents the contributions of each type of shock to the autocovariances while Figure 6 does the same for the cross covariances. For both output growth and inflation the autocovariances are dominated by permanent real supply shocks. Permanent nominal shocks also contribute substantially to positive autocovariance in inflation but tend to offset positive autocovariance in output growth. The autocovariances of interest rate changes are small relative to those of output growth and inflation (see Figure 3) and the sample values are positive at some lags, negative at others. The *estimated* autocovariances are all small and negative and are dominated at short lags by the real demand shock *r* (accommodated by monetary policy) and at longer lags by the permanent supply shock.

The lower half of Figure 3 shows that the largest cross covariances (in absolute value) are those involving output growth and lagged inflation. The model explains these reasonably well. On the other hand, the model fails to capture the change in sign on the cross covariances between output growth and *future* inflation. The decompositions in Figure 6 show that the correlations between output growth and future inflation are dominated by the two permanent shocks, which both affect the covariances negatively. On the other hand, a permanent nominal shock produces a positive covariance between output growth and lagged inflation, but not of sufficient magnitude to offset the negative covariances arising from the permanent supply shocks. Temporary demand shocks also affect the contemporaneous and once lagged covariance between output growth and inflation.

Figure 3 shows that the cross covariances between output growth and lagged changes in interest rates tend to be negative, those between output growth and *future* changes in interest rates positive. While the model generally captures the sign of these covariances, the magnitudes tend to be too small. Figure 6 shows that the effects on cross covariances of *both* permanent shocks change sign as we move from lags (greater than 1 in the case of permanent nominal shocks) to contemporaneous or leading covariances. Temporary real demand shocks are the largest contributor to a contemporaneous positive covariance between output growth and interest rate changes and also contribute to the negative covariances between current output growth and future interest rate changes.

Finally, Figure 3 reveals that the covariances between inflation and future interest rate changes tend to be small and change in sign, while the covariances between past interest rate changes and inflation tend to be positive but increase in size as the lag increases. Figure 6 reveals that the effects on cross covariances of *both* permanent shocks again change sign as we move from lags (greater than 2 in the case of permanent nominal shocks) to contemporaneous or leading covariances. Temporary real demand shocks are again important contributors to the contemporaneous and first order cross covariances.

8. Concluding remarks

This paper uses a method of moments procedure to estimate an aggregate demand/aggregate supply model with rational expectations. The empirical results suggest a significant role for rational expectations of future prices in macroeconomic fluctuations. However, the model was also based on an asymmetry between supply and demand behavior. Expectations relevant for the supply of output at *t* were assumed to be based on information available at $t-1$ whereas aggregate demand was assumed to depend on real interest rates based upon information available at t.

An advantage of our estimation procedure over structural VAR models is that we can have more shocks than endogenous variables. In addition to looking at whether shocks are predominantly supply or demand in origin, we allowed each type of shock to have separate permanent and temporary components. We also allowed temporary demand shocks to be either nominal or real. We found strong evidence for major roles in U.S. macroeconomic fluctuations for permanent supply, permanent nominal and temporary real demand shocks and less convincing evidence for temporary supply shocks.

The results indicate that the model can account for many features of the data with parameter estimates that appear reasonable. However, there are also some features of the data, and some parameter estimates, that indicate the model could be improved upon. Subject to this proviso, the model suggests, contrary to much of the real business cycle literature, that permanent nominal demand shocks arising from monetary policy or shifts in money demand play a dominant role in producing variance of U.S. output growth. On the other hand, the model also suggests a dominant role for permanent supply shocks in producing variance in inflation, serial correlation in both output growth and inflation and long run changes in the *level* of output. Temporary real shocks to demand are important in generating interest rate movements, and it appears that monetary policy tends to accommodate such shocks.

9. References:

Barro, Robert J. and David B. Gordon. "A Positive Theory of Monetary Policy in a Natural Rate Model." *Journal of Political Economy*, 1983, 91, 589-610.

Blanchard, Olivier J. and Danny Quah. "The Dynamic Effects of Aggregate Demand and Supply Disturbances." *American Economic Review*. 1989, 79, 655-73.

Blanchard, Olivier J. and Danny Quah. "The Dynamic Effects of Aggregate Demand and Supply Disturbances: Reply." *American Economic Review* 1993, 83, 653-658.

Blanchard, Olivier J. and Mark W. Watson. "Are Business Cycles All Alike?" In Robert J. Gordon, editor, *The American Business Cycle, Continuity and Change*. University of Chicago Press 1986, 123-179.

Brunner, K., A. Cukierman, and A.H. Meltzer, "Stagflation, Persistent Unemployment, and the Permanence of Economic Shocks." *Journal of Monetary Economics* 1980, 6, 467-492.

Buiter, Willem H., "Macroeconomic Policy During a Transition to Monetary Union." Centre for Economic Policy Research (London), Discussion Paper no. 1222 (1995).

Burnside, Craig and Martin Eichenbaum, "Small Sample Properties of Generalized Method of Moments Based Wald Tests," Working Paper, University of Pittsburgh and Northwestern University, 1994.

Campbell, John Y. and Pierre Perron. “Pitfalls and Opportunities: What Macroeconomists Should Know about Unit Roots.” In Olivier Jean Blanchard and Stanley Fischer, eds., *NBER Macroeconomics Annual*, Cambridge: MIT Press, 1991, 141-201.

Cooley, Thomas F., and Lee E. Ohanian, “The Cyclical Behavior of Prices.” *Journal of Monetary Economics* 1991, 28, 25-60.

Cumby, R.E., J. Huizinga and M. Obstfeld. “Two-Step Two-Stage Least Squares Estimation in Models with Rational Expectations.” *Journal of Econometrics.* 1983, 21, 333-55.

Engle, Robert F. and C.W.J. Granger. “Co-integration and Error Correction: Representation, Estimation, and Testing.” *Econometrica*. 1987, 55, 251-76.

Faust, Jon, and Eric Leeper. “When Do Long-Run Identifying Restrictions Give Reliable Results?” Board of Governors of the Federal Reserve System, *International Finance Discussion Paper No. 462*, March 1994.

Fischer, Stanley, “Long-term Contracts, Rational Expectations, and the Optimal Money Supply Rule.” *Journal of Political Economy* 1977, 85, 191-206.

Gali, Jordi. “How Well Does the IS-LM Model Fit Postwar U.S. Data?” *Quarterly Journal of Economics*. 1992, 107, 709-38.

Hansen, L.P. “Large Sample Properties of Generalized Method of Moments Estimators.” *Econometrica.* 1982, 50, 1029-54.

Hartley, Peter R., and Carl E. Walsh. “A Generalized Method of Moments Approach to Estimating a ‘Structural Vector Autoregression’.” *Journal of Macroeconomics.* 1992, 14, 199-232.

Hartley, Peter R., and Joseph A. Whitt Jr. “Macroeconomic Fluctuations in Europe: Demand or Supply, Permanent or Temporary?” Working Paper 97-14, Federal Reserve Bank of Atlanta, November 1997.

King, Robert G., Charles I. Plosser, James H. Stock, and Mark W. Watson. “Stochastic Trends and Economic Fluctuations.” *American Economic Review.* 1991, 81, 819-840.

Kwiatkowski, Denis, Peter C.B. Phillips, Peter Schmidt, and Yongcheol Shin. “Testing the null hypothesis of stationarity against the alternative of a unit root.” *Journal of Econometrics*, 1992, 54, 159-178.

Kydland, Finn E., and Edward C. Prescott. “Time to Build and Aggregate Fluctuations.” *Econometrica*. 1982, 50, 1345-1370.

Kydland and Prescott. “The Workweek of Capital and Its Cyclical Implications.” *Journal of Monetary Economics.* 1988, 21, 343-360.

Kydland, Finn E., and Edward C. Prescott, “Business Cycles: Real Facts and a Monetary Myth.” *Federal Reserve Bank of Minneapolis Quarterly Review*, Spring, 1990, 3-18.

Kydland and Prescott. “Hours and employment variation in business cycle theory.” *Economic Theory*. 1991, 1, 63-81.

Lippi, Marco and Lucrezia Reichlin. “The Dynamic Effects of Aggregate Demand and Supply Disturbances: Comment.” *American Economic Review.* 1993, 83, 644-652.

Long, John B. and Charles I. Plosser. “Real Business Cycles.” *Journal of Political Economy*. 1983, 91, 39-69.

Lucas, Robert E., Jr. “Some International Evidence on Output-Inflation Trade-offs.” *American Economic Review*. 1973, 63, 326-334.

McCallum, Bennett. *Monetary Economics*. New York: Macmillan, 1989.

McCallum, Bennett T. and Edward Nelson. “An Optimizing IS-LM Specification for Monetary Policy and Business Cycle Analysis.” mimeo, Carnegie Mellon University, December 1996.

Mackinnon, James G. “Critical Values for Cointegration Tests.” In R. F. Engle and C. W. J. Granger, editors, *Long-Run Economic Relationships, Readings in Cointegration*, Oxford: Oxford University Press, 1991, 267-276.

Newey, Whitney K., and Kenneth D. West. “A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix.” *Econometrica.* 1987, 55, 703-708.

Prescott, Edward C. “Response to a Skeptic.” *Federal Reserve Bank of Minneapolis Quarterly Review*, Fall 1986.

Rogoff, Kenneth, “The Optimal Degree of Commitment to an Intermediate Monetary Target.” *The Quarterly Journal of Economics*. 1985, 100, 1169-1189.

Shapiro, Matthew and Mark Watson. “Sources of Business Cycle Fluctuations.” In Stanley Fischer, ed., *NBER Macroeconomics Annual*, Cambridge: MIT Press, 1988, 111-48.

Sieper, E., “Policy Irrelevance Is Not the Rule”, mimeo, Australian National University, 1989.

Sims, Christopher A. “Macroeconomics and Reality.” *Econometrica*. 1980, 48, 1-48.