Macroeconomic Fluctuations: Demand or Supply, Permanent or Temporary?

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Abstract

We use generalized method of moments to estimate a rational expectations aggregate demand/aggregate supply macroeconomic model for five European economies and the United States. Our aim is to examine whether supply or demand shocks have predominated in these economies during the post-war era, and whether shocks of either type have been primarily temporary or permanent in nature. The estimation procedure is an alternative to estimating and interpreting vector autoregressions under restrictions of the Bernanke-Sims or Blanchard-Quah variety or to performing calibration exercises.

We find that all four types of shocks (permanent supply, permanent demand, temporary supply, and temporary demand) are needed to account for the data on output and inflation across all economies, although in a number of economies three shocks suffice. Permanent or temporary demand shocks have been the dominant source of variance in output growth in five of the six countries, but there is a less consistent pattern for inflation.

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1. Introduction

In recent years, a number of authors have used vector autoregressions (VAR's) to investigate whether macroeconomic fluctuations are primarily caused by nominal or real shocks. In this paper, we investigate the sources of macroeconomic fluctuations in the major European economies and the United States (US) by estimating an aggregate demand/aggregate supply model with rational expectations. Our model allows macroeconomic fluctuations to arise from either supply or demand shocks. We also allow the demand and supply shocks to have permanent and temporary components that are not separately identifiable. A distinctive feature of the analysis therefore is that the number of driving shocks exceeds the number of endogenous variables. Nevertheless, we are able to estimate the structural parameters, including the variances of the underlying shocks, using generalized method of moments.

Much of the literature on sources of macroeconomic fluctuations has used vector autoregressions (VAR's) rather than structural models. The classic paper by Sims (1980) found that nominal shocks were a major source of US fluctuations. Sims argued that the exclusion restrictions commonly used to identify parameters in traditional structural models were not reasonable under rational expectations. When expectations are rational, all relevant predictive variables belong in any equation where expectations appear. While a VAR treats all observable variables as endogenous, the parameter estimates are very difficult to interpret. As a substitute for exclusion restrictions, Sims assumed that his data could be ordered in a Wold causal chain. Since then, various other methods of identifying VAR's have been proposed.

Blanchard and Watson (1986) identify a VAR by restricting the contemporaneous correlations of the one-step-ahead forecast errors. They conclude that US fluctuations are due to fiscal, monetary, demand, and supply shocks, in roughly equal proportions.

Several other authors have used long-run restrictions to identify VAR's. After assuming that demand shocks have zero long-run impact on output, Blanchard and Quah (1989) find that demand shocks are the primary source of US fluctuations. By contrast, Shapiro and Watson (1988) find evidence that exogenous labor supply shocks drive U.S. fluctuations. King, Plosser, Stock, and Watson (1991), who use a combination of long and short-run restrictions to identify their VAR's, report that nominal shocks have little importance and find evidence of at least two separate real shocks.

Gali (1992) examines a structural VAR of the IS-LM variety for the US economy. He assumes there are four shocks: supply, money demand, money supply, and an IS shock (that is, three types of "demand" shocks, and one supply/productivity shock). He identifies parameters through a combination of long-run and short-run restrictions. He finds both types of shocks important, but supply shocks are dominant: 70

percent of output variability at business cycle frequencies is accounted for by supply shocks.

Long-run restrictions on VAR's of the Blanchard-Quah variety have also been used by Ahmed and Park (1994), Bergman (1996), Karras (1994), Bayoumi and Eichengreen (1992) and Whitt (1995) to examine evidence on the sources of macroeconomic shocks in other economies. Ahmed and Park focus on seven OECD countries, including five in Europe. They estimate VAR's with four endogenous variables: home country real output, the price level, the balance of trade, and rest-of-world output, proxied by U.S. output. They report strong support for one of the propositions of real-business-cycle theory, namely that supply-side changes explain the bulk of the movements in aggregate output. Bergman studies five countries, including Germany and the United Kingdom (UK), using a bivariate VAR model for output and inflation. Using variance decompositions, he argues that at a typical business cycle frequency (the five-year horizon), supply shocks are the main source of output variance for all his countries.[1]

By contrast, the other three papers find results less favorable to real-business-cycle theory. Karras (1994) estimates VAR's for three European countries, two of which (France and the UK) were analyzed by Ahmed and Park. He uses five variables: home country output, the price level, employment, the real interest rate, and the world price of oil. He concludes that real business cycle models are inadequate, because aggregate demand was responsible for over half of the variability of output at a four-quarter horizon in France and Germany, and about 40 percent in the UK. Bayoumi and Eichengreen (1992) and Whitt (1995) estimate VAR's with two variables, output and prices. Like Karras, they find that aggregate demand shocks account for a substantial portion of output fluctuations in major European countries.

Another strand of the literature on sources of macroeconomic fluctuations examines contemporaneous and lagged covariances of output and prices. Demand shocks are presumed to push output and prices in the same direction, while supply shocks push them in opposite directions. Traditionally, prices and output were thought to have positive covariances because demand shocks acting through some type of Phillips-curve relationship were presumed to dominate (see King and Watson (1994) for a review of econometric evidence on the Phillips curve). This view was challenged by Kydland and Prescott (1990 Table 4) and Cooley and Ohanian (1991 Table 1), who report that prices and output in the US have negative covariances at nearly all leads and lags. This would seem to imply that supply shocks are more important than demand shocks. However, Chadha and Prasad (1993) and Judd and Trehan (1995) show that for some common detrending methods, a model with only demand shocks can generate negative

1. It is debatable whether Bergman's results for Germany are entirely supportive of real-business-cycle theory. At a three-year (12-quarter) horizon, only 35 percent of output variance in Germany is attributable to supply shocks.

covariances between prices and output. In addition, den Haan (1996) develops descriptive statistics that (unlike unconditional covariances) are valid for both stationary and integrated series. He applies this method to US data and reports that in recent decades the co-movement of prices and output has been positive in the short run but negative in the long run, leading him to conclude that a plausible model would have demand shocks dominating in the short run, and supply shocks dominating in the long run.

In our data set (industrial production and producer prices), we found that sample covariances between output growth and lagged inflation were consistently negative for the US and five European countries. However, positive covariances between output growth and future inflation were seen in the US and three of the European countries.

We follow Hartley and Walsh (1992) and use a method of moments procedure to estimate the parameters of a small structural model of output fluctuations. An initial estimation chooses parameter values to minimize the sum of squared differences between the theoretical second moments implied by the structural model and the corresponding sample second moments obtained from the data. A second estimation minimizes a weighted sum of squared deviations with weights chosen "optimally" to yield a test of the parameter restrictions. Our results are immune from the Lippi and Reichlin (1993) and Faust and Leeper (1994) criticisms of the Blanchard-Quah approach. In addition, structural modeling of the type pursued in this paper gives estimated parameters that have a clear economic interpretation, something often lacking in VAR analyses.

To facilitate cross-country comparisons, we use the same simple structure for all countries, a variant of the model in Rogoff (1985) that has been used widely in analyses of optimal monetary policy.[2] We assume each country has an aggregate supply and an aggregate demand curve, each of which can be shifted by permanent or temporary shocks. Our model could be viewed as a stripped-down single-economy version of the multi-country structural model in Taylor (1993). Taylor's model has considerably more variables and sectoral detail than our model. It includes exogenous stochastic shocks to financial markets (such as the demand for money), goods markets (such as investment and the components of consumption), prices (such as wages and import prices), as well as monetary and fiscal policy shocks. Taylor does not provide a variance decomposition but does report that shocks to the demand for money (short-term interest rate), exchange rates, import prices, and certain components of aggregate demand (such as

2. Simple models of the type we examine in this paper can be criticised from the perspective of real business cycle theory for imposing too little structure, and from the perspective of VAR analyses for imposing too much structure. Since optimal policy under rational expectations is usually examined in the context of such simple aggregate supply/aggregate demand frameworks, however, it is useful to investigate how well such models can account for the regularities in the data.

durables consumption and inventory investment) tend to be large. Accordingly, his analysis provides information on the types of demand shocks that have been prevalent in the G–7 countries. An important difference between our model and Taylor's is that there are no stochastic supply shocks in Taylor's model, while we allow for both permanent and temporary supply shocks.[3]

The method of moments procedure is essential to enable us to estimate a model where the supply and demand curves are each affected by more than one stochastic shock. When the number of unobserved exogenous shocks exceeds the number of observed endogenous variables, the econometrician cannot recover time series for the shocks from the data, implying that the parameters cannot be estimated using maximum likelihood. However, the endogenous variables can be expressed as a vector autoregressive moving average process of the shocks. This VARMA representation yields expressions for the contemporaneous and lagged variance and covariances of the endogenous variables as a function of the various supply and demand elasticities and the variances of the underlying shocks. A disadvantage of our approach relative to analyses such as Taylor (1993) is that the "fundamental shocks" underlying our VARMA representation may be difficult to interpret in terms of observable factors like restrictions on energy supply, fiscal policy shocks or terms of trade shocks.

The method of moments estimation procedure can also be related to the "calibration method" used to evaluate real business cycle models. Both approaches attempt to explain the variances and covariances of key aggregate variables. In the real business cycle literature, however, model parameters are chosen to fit the *first* moments of the data and researchers then examine how well the model explains the second moments. In our case, parameter values are selected to match the second moments, which are then also used to evaluate the performance of the model. Since the number of moments that we seek to explain exceeds the number of parameters, however, we obtain a set of over-identifying restrictions that can be tested. The estimated standard errors obtained from the method of moments procedure also provide further information on the fit between the model and the data. Furthermore, the adequacy of the model can be judged on an informal basis by comparing many of our estimated parameter values with estimates obtained in other ways, including by fitting first moments.

We find that all four types of shocks (permanent supply, permanent demand, temporary supply, and temporary demand) are needed to account for the data. Permanent demand shocks have been the dominant source of *variance* in output growth in Germany, the UK, the US and the Netherlands. Temporary demand shocks have been about twice as important a source of variance in output growth in Italy, while

3. Taylor (1993, p97) specifies that potential output grows at a constant rate unaffected by policy or exogenous shocks.

all four shocks have been of roughly equal importance in France. With the caveat that the model usually had difficulty matching the autocovariances of output growth well, the results indicate that permanent supply shocks have been the main source of longer run positive *autocorrelation* in output growth in all countries. In all countries, demand shocks contribute to longer run negative autocorrelations in output growth.

Inflation variances and autocovariances have been dominated by permanent supply shocks in France. Permanent supply shocks also dominated in the US and UK, although permanent demand shocks were almost as important. In Germany and the Netherlands, permanent supply and demand shocks have been of roughly similar importance. In Italy, permanent demand shocks contributed most to the variance and autocovariances of inflation, although permanent supply shocks were also important.

In so far as the cross correlations are concerned, estimates from our model imply that permanent supply shocks tend to induce negative covariances between output growth and all leads and lags of inflation. However, temporary supply shocks, temporary demand shocks, and permanent demand shocks have more complicated effects because of the presence of lags and expectations in the model. For example, permanent demand shocks induce positive covariances between output growth and current and future inflation, but negative covariances between output growth and past inflation. In other words, the effects of permanent demand shocks on the correlations between output growth and inflation tend to counteract those of permanent supply shocks on one side of the correlogram while on the other side, they are reinforcing.

2. Integration and co-integration tests

There are few a priori theoretical restrictions on the possible number, or stationarity properties, of the shocks affecting the macroeconomy. Before developing and estimating the model, therefore, the data need to be examined for stationarity and possible co-integration features. The assumed stochastic structure of the theoretical model then needs to be consistent with the stationarity properties of the data.

Quarterly data on industrial production and producer prices, both seasonally-adjusted, were obtained from Haver for the United States and from the IFS or the BIS for the five largest West European economies – Germany, France, UK, Netherlands and Italy. The data are described in more detail in Appendix 1.

We chose industrial production rather than GDP to avoid the problems with measuring government output that infect the GDP statistics. It might also be reasonable to expect cycles in industrial production to be more similar across these economies than cycles in other components of output. As for prices, we reasoned that industrial output responds more directly to producer prices than to other indices such as consumer prices. Furthermore, a price index may be preferable to an implicit deflator for our purpose because measurement errors may by construction induce negative correlation between real GDP and the implicit deflator.

The well-known augmented Dickey-Fuller test was applied to the quarterly series, logged, in order to asses the number of unit roots (permanent shocks) in the data. The results are presented in Table 1.

TABLE 1. Dickey-Fuller tests of stationarity and co-integration[a]

	Industrial production		**Producer prices**		
	1 unit root[b]	**2 unit roots**	**1 unit root**	**2 unit roots**	**Co-integration**
Germany	-2.42 (-3.45)	-8.33	-1.31	-3.97	-1.87 (-3.86)
France	-3.54 (-3.46)	-8.14	-1.08	-4.72	-3.31 (-3.87)
UK	-2.89 (-3.44)	-10.76	-1.68	-2.32	-2.78 (-3.85)
Netherlands	-1.92 (-3.44)	-4.78	-1.22	-3.03	-1.60 (-3.85)
Italy	-1.95 (-3.45)	-11.90	-1.56	-4.90	-0.78 (-3.86)
US	–3.65 (–3.44)	–4.93 (–2.88)	–1.39 (–3.44)	–2.58 (–2.88)	

a. Tests were done including zero to eight lags of the dependent variables in order to deal with the possibility of serially-correlated residuals. The test statistics in the table use the specification that was the "best" according to the Schwarz criterion, subject to having residuals with a Ljung-Box Q statistic that failed to reject the null hypothesis of no serial correlation at a 10 percent or higher significance level.

b. The 5 percent critical value for each country's unit root tests are given in parentheses below the test statistics in column 1; the same critical value also applies to columns 2 to 4. The 5 percent critical values for the Engle-Granger co-integration tests are in parentheses below the test statistics in column 5. All critical values were obtained from Mackinnon (1991).

The pattern for Germany is clear: we fail to reject a single unit root in each series, we do reject two unit roots in each, and we fail to reject an absence of co-integration. For the other 25 tests, 20 conform to the German pattern.

The first exception is France (column 1). The test indicates a weak rejection of a single unit root in output (the 1 percent critical value is about -3.97), suggesting that the output level might be stationary. Similarly, the DF test rejects the presence of a unit root in US industrial production at the 5 percent level.[4]

The other three exceptions are in column 4 for the UK, the Netherlands and the US. In these instances, we fail to reject the null hypothesis of two unit roots in the price series at the five (or, for the UK and the

4. In the case of the US we also examined stationarity using the test developed by Kwiatkowski, Phillips, Schmidt and Shin (1992) (KPSS), which takes stationarity as the null. For industrial production in the US, the KPSS test rejected stationarity in the levels (at much below the 5 percent level) and failed to reject stationarity after first differencing.

Netherlands, even the 10) percent levels.[5] Graphs of the data for the UK and the Netherlands indicated that for a lengthy period in the middle of the sample (roughly 1973 to 1981), the mean rate of growth of prices was substantially higher than at other times.[6] We considered using dummy variables to create adjusted series, but chose not to do so for two reasons: first, such dummy variables might well remove from the data major supply or demand shocks, and second, we thought it desirable to maintain cross-country consistency by using the same pre-filter for all countries.

Based on these results, we constructed an aggregate demand/aggregate supply model with two independent permanent shocks.[7] We shall assume one of these shocks is a supply shock and one is demand shock. We make no assumptions, however, about whether the demand shock is real or nominal.

3. Aggregate Supply

We assume there is a supply shock s_t at t that is a *combined* temporary and permanent shock. Temporary supply shocks could represent the effect of strikes, severe weather or other temporary influences on aggregate production. Permanent supply shocks represent long-lasting shifts in aggregate supply associated, for example, with changes in technology and factor supplies. In practice, however, we do not attempt to identify any particular episodes as corresponding to our different types of shocks. Rather, our analysis can be viewed as asking whether fluctuations are primarily caused by "generic" supply or demand shocks and whether those shocks are primarily temporary or permanent in the way they are defined below.

While s_t is known at t, neither agents in the economy nor the observing econometrician know for sure what part of s_t will be permanent.[8] In our model, agents' confusion about whether shocks are permanent or temporary is important for generating macroeconomic fluctuations. For now, we assume that agents only learn the temporary versus permanent composition of supply shocks after one period.[9]

Following Rogoff (1985) and Barro and Gordon (1983), we assume that output supplied increases endog-

5. The KPSS test rejected stationarity of producer prices (again at much below the 5 percent level) and failed to reject stationarity for the inflation rate at the 5 percent, but not the 10 percent, level of significance.

6. A referee suggested that the evidence of non-stationarity in inflation in these economies might reflect changes in policy regime, particularly with regard to foreign exchange rates.

7. While aggregate demand/aggregate supply models that use the IS-LM framework have been criticized in recent years, McCallum (1989, p. 102-107) argues that if the supply function has classical properties, as is the case in the model in this paper, then the resulting model is for many purposes rather similar to models derived from explicit maximization of agents' choice problems.

8. Brunner, Cukierman and Meltzer (1980) developed a similar theoretical model in which macroeconomic fluctuations arise because agents cannot distinguish permanent from temporary shocks.

enously when current prices rise above the rationally expected prices based on the previous period's information. Lucas (1973) provides a justification for such an effect when suppliers are confused about whether shocks are primarily local (and real) or aggregate (and nominal). Our model does not distinguish between local and aggregate shocks, while agents always know the current demand and supply shocks. They are confused only about the permanence of those shocks. Nevertheless, we can obtain an analog of the Lucas supply curve if we assume suppliers base their expectations on last period's information. Alternatively, Fischer (1977) generates such a supply curve in a model where suppliers pre-commit to contracts one period in advance.

In a departure from Rogoff (1985), we allow supply to be autocorrelated. This could result, for example, from investments that transmit current deviations of supply into future periods. Thus, the aggregate supply curve can be written (where all variables are in logarithms):

$$y_t = \rho y_{t-1} + \gamma(p_t - E_{t-1}p_t) + s_t \tag{1}$$

with

$$s_t - s_{t-1} = s_t^P + s_t^T - s_{t-1}^T \;. \tag{2}$$

The shocks s_t^P (the innovation to the permanent component of the overall supply shock) and s_t^T (the temporary component) are assumed to be uncorrelated at all leads and lags and each of them is assumed to be independently identically distributed. Because we use GMM for estimation, we do not need to specify a distribution for the shocks s_t^P and s_t^T. We merely need to assume that both shocks have finite second moments. The same is true of the components of the demand shocks that are specified below.

We shall assume that the number of integrated random variables among the driving shocks matches the number of non-stationary driving shocks indicated by the unit-root and co-integration analysis. The structural model then must be constructed so that it would yield stationary endogenous variables if the driving shocks had also been stationary. In particular, the autocorrelation parameter, ρ needs to lie in the interval (-1,1). We expect the elasticity coefficient γ to be positive.

4. Aggregate Demand

Aggregate demand is assumed to reflect intertemporal substitution and, if substitution effects dominate,

9. As a referee noted, the assumption that uncertainty about the permanence of shocks is resolved after one period is critical in keeping the model tractable. Allowing agents to learn about shock permanence through some kind of filter would greatly complicate the analysis. Since we can readily extend the model to any finite number of periods of uncertainty, we check the robustness of our results to this assumption by considering a model where agents do not know the composition of the supply and demand shocks for two periods.

to respond negatively to the current real interest rate. As in Rogoff (1985), we assume that, in contrast to factor markets where expectations are based on information available at t-1, expectations in capital markets are based on information available at t. We also allow aggregate demand to be autocorrelated.

There is also a real demand shock χ_t that represents shifts in the IS curve. Examples of such shifts are changes in demographics, fiscal policy or export demand. As with aggregate supply, we allow aggregate demand to be autocorrelated. Thus, the aggregate demand curve can be written (with variables other than the interest rate in logarithms):

$$y_t = \eta y_{t-1} - \alpha(i_t - E_t p_{t+1} + p_t) + \chi_t \,. \tag{3}$$

We expect α to be positive and again require the autocorrelation parameter, η to lie in the interval (-1,1).

Money Market

We also postulate a conventional aggregate demand for money balances:

$$m_t - p_t = \beta y_t - \delta^{-1} i_t + \omega_t \tag{4}$$

where ω is a shock to money demand and δ^{-1} is the interest semi-elasticity of the demand for money.

Reduced form aggregate demand curve

We assume equilibrium p_t and i_t equate aggregate supply and aggregate demand for goods and money. From the money market equilibrium condition we can conclude that

$$i_t = \beta\delta y_t - \delta(m_t - p_t - \omega_t) \,. \tag{5}$$

Substitute (5) into the aggregate demand curve (3) to deduce that it can be written:

$$y_t = \eta y_{t-1} - \alpha\left[\beta\delta y_t - \delta\left(m_t - \omega_t + \frac{\chi_t}{\alpha\delta} - p_t\right) - (E_t p_{t+1} - p_t)\right] \tag{6}$$

Equation (6) can be re-arranged to yield

$$y_t = \frac{\eta}{1+\alpha\beta\delta} y_{t-1} + \frac{\alpha\delta}{1+\alpha\beta\delta}\left(m_t - \omega_t + \frac{\chi_t}{\alpha\delta} - p_t\right) + \frac{\alpha}{1+\alpha\beta\delta}(E_t p_{t+1} - p_t) \tag{7}$$

As shown in the middle term in (7), shocks to aggregate demand can arise in many ways: besides the real demand (IS) shocks represented by χ_t, monetary policy can generate nominal shocks by changing the true money supply m, and changes in financial intermediation technology among other factors can produce real shocks to money demand ω.

We assume[10] neither the public nor the econometrician observe m, ω or χ. Nevertheless, using y, p, $E_t p_{t+1}$

and (7) the public can infer the value of the amalgamated demand shock d (defined as $m_t - \omega_t + \chi_t / \alpha\delta$). We can write the aggregate demand curve in terms of prices and the demand shock d alone in the form

$$y_t = \psi y_{t-1} + \Gamma(d_t - p_t) + \Phi(E_t p_{t+1} - p_t) \tag{8}$$

We conduct the subsequent analysis using (8) for the aggregate demand curve. As with much of the previous literature that has examined whether shocks to the economy are primarily supply or demand in character, we thus do not further separate demand shocks into nominal and real components. Moreover, possible endogenous policy responses to other shocks that hit the economy may be a problem with all attempts to distinguish whether shocks to the economy have primarily been "supply" or "demand" in character. We plan to address these limitations of the current analysis in future work.

Analogously to the supply shock s_t we assume that the demand shock d_t is a *combined* temporary and permanent shock.

$$d_t - d_{t-1} = d_t^P + d_t^T - d_{t-1}^T \tag{9}$$

The shocks d_t^P and d_t^T are assumed to be independently identically distributed and uncorrelated at all leads and lags with each other and with the supply shocks.

We again assume that neither the econometrician nor the agents in the economy know how much of a current demand shock is temporary and how much is permanent. Specifically, while d_t is known, the components d_t^P and d_t^T are not. We again assume, however, that agents learn the temporary versus permanent (but not the real versus nominal) composition of d_t after one (or, later in the paper, two) period(s).

5. Equilibrium

Using the lag operator L, the aggregate supply curve (1) can be written:

$$(1 - \rho L)y_t = \gamma(p_t - E_{t-1}p_t) + s_t \tag{10}$$

while the aggregate demand curve (8) can be written

$$(1 - \psi L)y_t = \Phi E_t p_{t+1} - (\Phi + \Gamma)p_t + \Gamma d_t . \tag{11}$$

Multiplying (10) by $(1-\psi L)$ and (11) by $(1-\rho L)$ we deduce that product market equilibrium requires

[10] A referee questioned the assumption that m_t cannot be observed on the grounds that statistics on the money supply are readily available. While this is true, the sheer variety of published aggregates, and the debates over which (if any) should be targeted by the monetary authority, suggest that even specialists find it difficult to agree on what the relevant monetary aggregate is. In addition, the full "monetary" shock to aggregate demand is $m_t - \omega_t$ and while the money supply might be observable, money demand shocks are much less so. In future work we plan to extend the current framework by allowing the public to observe more variables than just prices and output..

$$(1-\psi L)[\gamma(p_t - E_{t-1}p_t) + s_t] = (1-\rho L)[\Phi E_t p_{t+1} - (\Phi+\Gamma)p_t + \Gamma d_t]$$
$$= \Phi E_t p_{t+1} - \Phi\rho E_{t-1}p_t - (\Phi+\Gamma)(1-\rho L)p_t + \Gamma(1-\rho L)d_t \tag{12}$$

Since the composite shocks s_t and d_t are non-stationary, p_t is also non-stationary. To solve for the equilibrium price and output, we need to manipulate equation (12) to ensure we are working in spaces of stationary processes. By adding and subtracting $\Phi\rho p_t$, equation (12) can be re-arranged to obtain

$$\Phi E_t p_{t+1} - (\Phi+\Gamma)(1-\rho L)p_t - \Phi\rho p_t = (1-\psi L)s_t - \Gamma(1-\rho L)d_t + (\gamma - \Phi\rho - \psi\gamma L)(p_t - E_{t-1}p_t). \tag{13}$$

Now observe that $p_t - E_{t-1}p_t = P_t - E_{t-1}P_t$ is stationary[11] while

$$(1-L)E_t p_{t+1} = E_t p_{t+1} - E_{t-1}p_t = E_t p_{t+1} - p_t + p_t - E_{t-1}p_t = E_t[(1-L)p_{t+1}] + (p_t - E_{t-1}p_t). \tag{14}$$

Thus, differencing (13), we obtain a stochastic difference equation for $P_t = (1\text{-}L)p_t$:

$$\Phi E_t P_{t+1} - (\Phi+\Gamma)(1-\rho L)P_t - \Phi\rho P_t = (1-\psi L)(s_t^P + s_t^T - s_{t-1}^T) - \Gamma(1-\rho L)(d_t^P + d_t^T - d_{t-1}^T) +$$
$$[(\gamma - \Phi\rho - \Phi) - (\psi\gamma + \gamma - \Phi\rho)L + \psi\gamma L^2](P_t - E_{t-1}P_t) \tag{15}$$

6. Information processing

Individuals know the functional forms of the aggregate demand and supply curves. They also know p_t and y_t, and therefore the values of s_t and d_t, at time t. We assume to begin with, however, they do not know the decomposition of s_t or d_t into their components s_t^P, s_t^T, d_t^P or d_t^T until period t+1. From these assumptions about information, and the form of (15), we deduce that P_t will be a linear function of current and lagged s_t^P, s_t^T, d_t^P and d_t^T. Since individuals know, at t-1, all shocks dated t-2 or earlier, $(P_t - E_{t-1}P_t)$ will be a linear sum:

$$P_t - E_{t-1}P_t = \pi_{10}s_t^P + \pi_{20}s_t^T + \pi_{30}d_t^P + \pi_{40}d_t^T + \pi_{11}(s_{t-1}^P - E_{t-1}s_{t-1}^P) + \pi_{21}(s_{t-1}^T - E_{t-1}s_{t-1}^T)$$
$$+ \pi_{31}(d_{t-1}^P - E_{t-1}d_{t-1}^P) + \pi_{41}(d_{t-1}^T - E_{t-1}d_{t-1}^T) \tag{16}$$

Since individuals know $\Delta s_{t-1} = s_{t-1}^P + s_{t-1}^T - s_{t-2}^T$, $\Delta d_{t-1} = d_{t-1}^P + d_{t-1}^T - d_{t-2}^T$, s_{t-2}^P, s_{t-2}^T, d_{t-2}^P and d_{t-2}^T at t-1, they will also observe $s_{t-1}^P + s_{t-1}^T$ and $d_{t-1}^P + d_{t-1}^T$. Projecting onto these variables they would obtain:

$$E_{t-1}s_{t-1}^P = a_1(s_{t-1}^P + s_{t-1}^T) \tag{17}$$

$$E_{t-1}s_{t-2}^T = a_2(s_{t-1}^P + s_{t-1}^T) \tag{18}$$

$$E_{t-1}d_{t-1}^P = b_1(d_{t-1}^P + d_{t-1}^T) \tag{19}$$

$$E_{t-1}d_{t-1}^T = b_2(d_{t-1}^P + d_{t-1}^T) \tag{20}$$

[11] Thus, while p_t and $E_{t\text{-}1}p_t$ are both non-stationary, they are co-integrated.

where

$$a_1 = \frac{\sigma^2_{s^P}}{\sigma^2_{s^P} + \sigma^2_{s^T}},\ a_2 = 1 – a_1,\ b_1 = \frac{\sigma^2_{d^P}}{\sigma^2_{d^P} + \sigma^2_{d^T}} \text{ and } b_2 = 1 – b_1. \tag{21}$$

7. ARIMA representations for p_t and y_t

We define the inverse of the lag operator by

$$L^{-1}x_{t-i} = \begin{cases} x_{t-i+1} & i > 0 \\ E_t x_{t-i+1} & i \le 0 \end{cases} \tag{22}$$

where x_t is known at time t. Then the equilibrium solution for P_t can be written in terms of current and lagged shocks using the operators L and L^{-1}:

Lemma 1: The equilibrium inflation rate P_t satisfies the stochastic difference equation:

$$(\Phi+\Gamma)\left(1-\frac{\Phi}{\Phi+\Gamma}L^{-1}\right)(1-\rho L)P_t = \Gamma(1-\rho L)(d_t^P + d_t^T - d_{t-1}^T) - (1-\psi L)(s_t^P + s_t^T - s_{t-1}^T) - \tag{23}$$

$$[(\gamma - \Phi\rho - \Phi) - (\psi\gamma + \gamma - \Phi\rho)L + \psi\gamma L^2]\left[\sum_{i=0}^{1}(\kappa_{1i}s_{t-i}^P + \kappa_{2i}s_{t-i}^T + \kappa_{3i}d_{t-i}^P + \kappa_{4i}d_{t-i}^T)\right]$$

for constant coefficients $\kappa_{i0} = \pi_{i0}$, $i = 1,\ldots,4$, and

$$\kappa_{11} = (\pi_{11} - \pi_{21})a_2,\ \kappa_{21} = -(\pi_{11} - \pi_{21})a_1,\ \kappa_{31} = (\pi_{31} - \pi_{41})b_2 \text{ and } \kappa_{41} = -(\pi_{31} - \pi_{41})b_1. \tag{24}$$

Proof. The left side of (15) can be written

$$\Phi E_t P_{t+1} - (\Phi+\Gamma)(1-\rho L)P_t - \Phi\rho P_t = -(\Phi+\Gamma)\left[-\frac{\Phi}{\Phi+\Gamma}E_t P_{t+1} + \left(1+\frac{\Phi}{\Phi+\Gamma}\rho\right)P_t - \rho P_{t-1}\right] \tag{25}$$

$$= -(\Phi+\Gamma)\left[\left(1-\frac{\Phi}{\Phi+\Gamma}L^{-1}\right)P_t - \left(1-\frac{\Phi}{\Phi+\Gamma}L^{-1}\right)\rho P_{t-1}\right]$$

Also, substitute (17)–(20) into the right side of (16) and then substitute the result into (15).

Now define $F = \Phi/(\Phi+\Gamma) = 1/(1+\delta)$ and observe that unless $–2 < \delta < 0$, $|F| < 1$. Also, all the shocks on the right side of (23) are stationary. The polynomial in L^{-1} on the left side of (23) can therefore be expanded as a geometric series on the right side of (23). Then by using (22), and the fact that the shocks on the right side of (23) are independently distributed we can show:

Theorem 1: When the composition of shocks is unknown for one period, equilibrium inflation satisfies

$$(1-\rho L)P_t = \sum_{i=0}^{3}[\pi_{1i}s^P_{t-i} + \pi_{2i}s^T_{t-i} + \pi_{3i}d^P_{t-i} + \pi_{4i}d^T_{t-i}] \tag{26}$$

for constant coefficients π_{ij}, i = 1,…,4, j = 0,…,3 that depend on the elasticities, the shock variances and the information processing coefficients (21).

Proof. The proof is given in a technical appendix available from the authors.[12]

Comment: Note that the solution (26) is consistent with the unanticipated inflation rate given in (16).

Use Π_1 for the 4×4 matrix of MA coefficients with Π_{1j} the jth column of Π_1, so the 4 polynomials multiplying $z_t' = \left[s_t^P\ s_t^T\ d_t^P\ d_t^T\right]$ are the rows of

$$\Pi_1(L) = \sum_{j=1}^{4}\Pi_{1j}'L^{j-1}$$

Then we can write the ARMA(1,3) representation for P_t as:

$$(1-\rho L)P_t = \Pi_1(L)z_t\,. \tag{27}$$

From the supply curve (1), (16) and (17)–(20) we obtain an expression for equilibrium output:

Theorem 2: When the composition of shocks is unknown for one period, equilibrium output y_t satisfies:

$$(1-\rho L)y_t = s_t + \gamma\sum_{i=0}^{1}(\kappa_{1i}s^P_{t-i} + \kappa_{2i}s^T_{t-i} + \kappa_{3i}d^P_{t-i} + \kappa_{4i}d^T_{t-i}) \tag{28}$$

where $\kappa_{i0} = \pi_{i0}$, i = 1,…4 while κ_{i1}, i = 1,…4, satisfy (24).

Proof. Substitute (17)–(20) and the right hand side of (16) into the aggregate supply curve (1).

Corollary: The first difference of the equilibrium output $Y_t = \Delta y_t$ follows an ARMA(1,2) process.

Proof. Multiply (28) through by (1–L).

If we define a 4×3 matrix Π_2 of MA coefficients, we can write the ARMA(1,2) representation for Y_t:

$$(1-\rho L)Y_t = \Pi_2(L)z_t\,. \tag{29}$$

the technical appendix shows how manipulations that are by now quite standard in the time series literature can be used along with the ARMA representations (27) for $P_t = \Delta p_t$ and (29) $Y_t = \Delta y_t$ to derive theoretical expressions for the variances and autocovariances of P_t or Y_t and the cross covariances between current and lagged values of P_t and Y_t.

12. The appendix is also available at the Federal Reserve Bank of Atlanta world wide web site http://www.atl.frb.org/

8. Estimating the parameters using GMM

We examined lags up to six quarters for the autocovariances and cross covariances. We expected that this would cover a substantial part of typical cyclical fluctuations while leaving us a reasonable sample size (from the original roughly 100 to 130 quarters). We thus obtained theoretical expressions for 2 variances and 25 covariances of rates of change of equilibrium output and price. There are 9 parameters in these expressions. We can write the vector of parameters to be estimated as[13]

$$b = [\rho, \psi, \gamma, \Gamma, \Phi, \sigma_{s^P}, \sigma_{s^T}, \sigma_{d^P}, \sigma_{d^T}] \tag{30}$$

and we can denote the 27×1 vector of theoretical second moments by $\theta(b)$.

From the data, we have N observations on trend-corrected and seasonally adjusted quarterly rates of change in industrial production and producer prices. Using this data, we calculate $27 \times N$ cross products corresponding to our 27 theoretical second moments, with one set of cross products for each period n. Let $f(\Delta x_n, b)$ denote the 27×1 vector of differences between the sample cross products in period n and the corresponding theoretical second moments in $\theta(b)$. Under the null hypothesis, $E[f(\Delta x_n, b)] = 0$. If we form

$$g_N(b) = \frac{1}{N} \sum_{n=1}^{N} f(\Delta x_n, b)\,, \tag{31}$$

initial estimates $\hat{b}$ of b can be obtained by minimizing the sum of squared errors $g_N(b)'g_N(b)$.[14] The optimal GMM estimator (in the sense that the asymptotic covariance matrix of b is as small as possible) is then obtained by minimizing a weighted sum of squares[15] $g_N(b)'Wg_N(b)$, for a symmetric weighting matrix W which is a consistent estimator of S^{-1} where the matrix S is defined by

$$S = \sum_{j=-\infty}^{\infty} E[f(\Delta x_0, b) f(\Delta x_{-j}, b)']\,. \tag{32}$$

If we let $\tilde{b}$ be the parameter vector which minimizes this weighted sum of squares then $\sqrt{N}(\tilde{b} - b)$ will converge in distribution to a random vector with mean zero and covariance matrix $(D'SD)^{-1}$ where

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13. We estimated standard deviations instead of variances to impose the restriction that the variances are non-negative.

14. In practice, the numerical minimization algorithm worked better when we normalized by re-scaling parameter values and dividing $g_N(b)'g_N(b)$ by the sum of squared values of the sample moments. We used a combination of a derivative-based quasi-Newton method and the Nelder-Mead simplex algorithm to minimize the highly non-linear objective function. The simplex algorithm proved more effective at finding the general region of parameter space where a minimum lies, while the derivative-based algorithm was more effective at actually attaining the local minimum to be found in that region. To ensure we obtained a global minimum of the objective function, we tried many different starting values for the parameters.

15. In effect, the weighting matrix emphasizes those moments that can be estimated more precisely from the data.

$$D = E\left[\frac{\partial}{\partial b} f(b)\right]. \tag{33}$$

Following Newey and West (1987) we estimate S in (32) by[16]

$$\hat{S}_J = \hat{\Omega}_0 + \sum_{j=1}^{J} w(j,J)[\hat{\Omega}_j + \hat{\Omega}_j'] \tag{34}$$

where $w(j,J) = 1-[j/(J+1)]$ is a linearly declining weighting function and

$$\hat{\Omega}_j = \frac{1}{N} \sum_{n=j+1}^{N} f(\Delta x_n, b) f(\Delta x_{n-j}, b)' . \tag{35}$$

We also estimate D in (33) by

$$\hat{D} = \left[\frac{\partial}{\partial b} g_N(\hat{b})\right] = -\left[\frac{\partial}{\partial b} \theta(\hat{b})\right] \tag{36}$$

and the limiting covariance matrix of $\sqrt{N}(\tilde{b}-b)$ by

$$(\hat{D}'\hat{S}_J\hat{D})^{-1}. \tag{37}$$

The over-identifying parameter restrictions can are tested by evaluating

$$Ng_N(\tilde{b})'(\hat{S}_N)^{-1} g_N(\tilde{b}) , \tag{38}$$

which converges in distribution to a chi-square random variable with $r-q$ degrees of freedom where r is the number of moment conditions (27 in our case) and q the number of parameters (9 in our case).

By analogy with variance decompositions in VAR’s, we use the final parameter estimates to decompose the variances and covariances into the components due to each of the underlying shocks. This will provide our measure of the relative importance of supply and demand, and temporary and permanent shocks (as we have defined them) in driving output and prices over the sample period.

Identification of the parameters is fundamentally based on an assumptions that shocks to supply are orthogonal to shocks to demand and that the temporary components of each type of shock, and the innovations to the permanent components, are independently identically distributed. Our parameter estimates also depend on the functional forms of the supply and demand curves, our assumptions about the information available to agents in the economy, and the hypothesis that expectations are formed rationally.

16. In our empirical analysis, we used $J = 12$.

9. Results for the first model

The 27 moments used to estimate the model were the variance of output growth, the contemporaneous covariance between output growth and producer-price inflation, the variance of inflation, each variable's autocovariances up to six quarters, the contemporaneous cross-covariance, and other cross-covariances going forward and back up to six quarters. In all countries the sample variance of output growth is greater than the variance of inflation, but the disparity varies considerably across countries. The ratio of the variance of output growth to the variance of inflation ranges from 4.2 for Germany to 1.4 for Italy.

The pattern of sample cross-covariances warrants discussion. Kydland and Prescott (1990) and Cooley and Ohanian (1991) report negative cross-covariances between filtered prices and output for the United States at nearly all leads and lags. This led Kydland and Prescott to call the notion of a positive relationship between prices and output a monetary myth.

For our countries, we find somewhat different patterns. The contemporaneous cross-covariance is sizeable and negative for the UK and the US, but small and positive for France and Italy, and small and negative for Germany and Netherlands. The cross-covariances between output growth and positive lags of inflation are consistently negative, thereby conforming to the pattern reported by Kydland and Prescott (1990) and Cooley and Ohanian (1991): the negative sign means that when inflation rises, output tends to fall several quarters later. However, the cross-covariances in the other direction, between output growth and future (negative lags of) inflation, are quite variable: mostly negative for the UK and the Netherlands, negative at short leads and positive at longer leads for the US, and mostly positive for the other three countries.[17]

The least squares estimates of the parameters, and the corresponding minimized value for the (normalized) sum of squares objective function, are presented in Table 2. We defined the parameters so that all except the autocorrelation coefficients (ρ and ψ) should be positive. If ρ represents lags in the capital accumulation process, however, we would expect it also to be positive. We do require both ρ and ψ to be less than 1 in absolute value. As with ARIMA models, the same autocorrelation structure can be explained either by stationary or non-stationary, and invertible or non-invertible processes. We have eliminated this identification problem by ensuring the numerical algorithm concentrates on stationary and

17. Several factors may account for the differences between our results and those of Cooley and Ohanian and Kydland and Prescott: the countries, the measures of output and inflation, and the way the data were filtered all differ. Cooley and Ohanian use real GNP and implicit price deflators, while Kydland and Prescott use real GNP and two price measures, the implicit price deflator and consumer price. We use industrial production and producer prices. As for filters, Kydland and Prescott use only the Hodrick-Prescott filter, while Cooley and Ohanian use three filters: linear detrending, differencing, and the Hodrick-Prescott filter. We use differencing but in addition we remove a linear trend and seasonal effects.

invertible representations of the data. Similarly, the coefficient F on the forward operator is required to be less than 1 in absolute value.[18]

We normalized the sum of squared differences between the sample and theoretical second moments by dividing by the sum of the squared second moments. The least squares objective function can thus be thought of as a type of R^2 measure. It tells us the proportion of the "variation" in the second moments that the theoretical model explains. Except for France, the estimated model accounts for over 90 percent of the variability of the 27 moments.

TABLE 2. Least squares parameter estimates

Parameter	**Germany**[a]	**France**	**U.K.**	**Netherlands**	**Italy**	**U.S.**
$\tanh^{-1}(\rho)$	1.1799	1.2438	1.5784	1.7080	1.2144	1.1127
$\tanh^{-1}(\psi)$	0.2483	0.4219	-0.0136	0.3251	0.0198	0.1055
γ	3.5385	2.1389	9.2372	3.7430	2.1372	2.8985
Γ	1.1034	0.2943	0.4533	0.4877	0.6219	0.8770
$\tanh^{-1}(F)$	0.4600	1.4696	-2.9552	1.6942	-3.0595	0.2833
σ_{s^P}	0.004694	0.002508	0.001812	0.001397	0.003311	0.00516
σ_{s^T}	0.015243	0.016526	0.060779	0.035365	0.029791	0.01023
σ_{d^P}	0.014690	0.000010	0.043592	0.009277	0.056177	0.01781
σ_{d^T}	0.0000003	0.002442	0.000323	0.011721	0.000050	0.000035
LS objective	0.09181	0.13900	0.02689	0.05300	0.08444	0.12788

a. We found a second set of estimates for Germany with a slightly smaller least squares value, but it produced a larger weighted least squares objective and somewhat less satisfactory least squares estimates. The alternative model assigned a smaller role to temporary supply shocks and a larger role to temporary demand shocks. It also produced higher values for ρ, ψ and γ, and lower values for Γ and *F*. We have reported only the second set of estimates to save space, although interested readers can obtain the estimates for the other model from http://www.atl.frb.org/.

The minimized value for the least squares objective function was lowest for the UK, highest for France. One might conclude that the model performs best for the UK and worst for France, with the other countries in between. Such a conclusion would be unwarranted, however, since the least squares objective function is not the best measure of the fit between the theoretical model and the data. The least squares objective function, in common with "calibration" exercises, places greater weight on explaining the larger moments (in absolute value). By contrast, the weighted least squares procedure places greater weight on explaining the moments that can be estimated more precisely from the data in the sense that they have a lower sample variance.

The weighted least squares estimates, together with their standard errors estimated according to (37), are presented in Table 3. In all countries, the minimized weighted least squares objective function (38) was

18. We estimated the inverse hyperbolic tangents of $F = \Phi/(\Phi+\Gamma)$, ρ and ψ to impose the conditions $|F|<1$, $|\rho|<1$ and $|\psi|<1$.

well below conventional significance levels for a chi-squared random variable with 18 degrees of freedom. However, the distribution of this statistic in samples as small as ours is unlikely to be chi-squared with the hypothesized degrees of freedom.[19]

TABLE 3. Weighted least squares parameter estimates[a]

Parameter	Germany[b]	France	U.K.	Netherlands	Italy	U.S.
$\tanh^{-1}(\rho)$	1.1866 (0.0802)	1.2842 (0.0974)	1.5938 (0.1339)	1.6603 (0.2964)	1.2901 (0.1055)	1.1812 (0.0873)
$\tanh^{-1}(\psi)$	0.1881 (0.1279)	0.4084 (1.3584)	-0.0105 (0.0919)	0.2798 (0.2348)	0.0203 (0.1370)	0.1215 (0.0423)
γ	3.6006 (0.4013)	2.1817 (0.9649)	10.3710 (2.6864)	3.5410 (0.7647)	2.2602 (0.2764)	2.9043 (0.2589)
Γ	1.1765 (0.1940)	0.3304 (0.6180)	0.4676 (0.0924)	0.5099 (0.3868)	0.5810 (0.3077)	0.8373 (0.1571)
$\tanh^{-1}(F)$	0.2049 (0.6907)	1.3701 (0.5966)	-2.8535 (1276.6)	1.6610 (0.6960)	-2.2777 (338.12)	0.3126 (0.3042)
σ_{s^P}	0.004467 (0.00043)	0.002437 (0.00028)	0.001746 (0.00067)	0.001369 (0.00075)	0.003001 (0.00054)	0.004458 (0.00103)
σ_{s^T}	0.015464 (0.001709)	0.015551 (0.00518)	0.062807 (0.01622)	0.033528 (0.00557)	0.02950 (0.00282)	0.010811 (0.00177)
σ_{d^P}	0.015676 (0.002957)	0.010426 (0.01003)	0.039376 (0.01661)	0.010572 (0.01860)	0.053405 (0.00815)	0.016556 (0.00226)
σ_{d^T}	1.4e-08 (6.6480)	0.004775 (0.35501)	0.000002 (1.33870)	0.000059 (0.74565)	0.000014 (1.7011)	2.141e-07 (0.57592)
χ^2 (18 d.f.) (P-value)	9.372 (0.951)	6.946 (0.991)	8.120 (0.977)	9.799 (0.938)	8.709 (0.966)	9.1907 (0.955)
Implied parameter values[c]						
ρ	0.8301	0.8576	0.9207	0.9303	0.8591	0.8278
ψ	0.1859	0.3871	-0.0105	0.2727	0.0203	0.1209
F	0.2021	0.8787	-0.9934	0.9304	-0.9792	0.3028
Φ	0.298	2.394	-0.233	6.812	-0.2874	0.3636
δ^{-1}	0.253	7.246	-0.498	13.359	-0.495	0.4342
$\beta+\delta^{-1}\alpha^{-1}$	0.850	3.027	2.139	1.961	1.721	

a. Standard errors are in parentheses below each parameter estimate.

b. The second set of estimates for Germany produced a χ^2 of 9.447 and an estimate of $\tanh^{-1}(F)$ of –0.0379 with a standard error of 16.404. The estimated standard errors on $\tanh^{-1}(\psi)$, γ, Γ and σ_{s^T} were also large (4.77, 18.09, 5.87 and 76.667 respectively).

c. The income elasticity of money demand β and the real interest elasticity of demand α cannot be recovered.

While the overall fit appears good, some of the estimated parameter values do not accord with our prior expectations, notably the negative estimates of F (and hence Φ) for Italy and the UK. Also, some of the estimated standard errors are large.

19. Burnside and Eichenbaum (1994) examine the small sample properties of GMM estimators.

While unexpected parameter estimates, or large standard errors, may indicate an inadequate theoretical framework, other factors might also be relevant. While many lag structures could be consistent with the basic theoretical framework, we did not adjust the lags in the model to better fit the data.[20] Also, the lack of a specified distribution for the shocks may have reduced our ability to obtain tightly estimated standard errors. Finally, many of the parameters are unlikely to have been constant over the sample period. We are not estimating “deep structural parameters” (arising from a specification of relatively stable taste and technology functions), and policies and other sources of shocks are likely to have varied over time.

10. An alternative information assumption

A change in the amount of information available to individuals substantially alters equilibrium prices and output. To illustrate this, we now assume that agents do not know the decomposition of s_t or d_t into their components s_t^P, s_t^T, d_t^P or d_t^T until period t+2.

Theorem 3: When the composition of shocks is unknown for two periods, equilibrium inflation satisfies

$$(1-\rho L)P_t = \sum_{i=0}^{4}[\pi_{1i}s_{t-i}^P + \pi_{2i}s_{t-i}^T + \pi_{3i}d_{t-i}^P + \pi_{4i}d_{t-i}^T] \tag{39}$$

where the 18 distinct coefficients π_{ij}, $i = 1,\ldots,4$, $j = 0,\ldots,4$ (with $\pi_{10} = \pi_{20}$ and $\pi_{30} = \pi_{40}$) satisfy 18 simultaneous equations that depend on the elasticities, the shock variances and an expanded set of information processing coefficients.

Proof. The proof is given in the technical appendix available from the authors.

Thus, under the modified information assumptions, P_t follows an ARMA(1,4) process. From the supply curve (1) and (39) we obtain an expression for equilibrium output:

Theorem 4: When the composition of shocks is unknown for two periods, equilibrium output y_t satisfies:

$$(1-\rho L)y_t = s_t + \gamma\sum_{i=0}^{2}(\kappa_{1i}s_{t-i}^P + \kappa_{2i}s_{t-i}^T + \kappa_{3i}d_{t-i}^P + \kappa_{4i}d_{t-i}^T) \tag{40}$$

where $\kappa_{i0} = \pi_{i0}$, $i = 1,\ldots 4$ while κ_{ij}, $i = 1,\ldots 4$, $j = 1,2$ satisfy a system of simultaneous equations involving the π_{ij} and the information processing coefficients.

Proof. The proof is given in the technical appendix available from the authors.

Multiplying (40) through by (1–L), we conclude that, under the modified information assumptions, the

[20]. In many studies, lag lengths are chosen ex-post using the Akaike or a similar goodness-of-fit criterion.

first difference of equilibrium output $Y_t = \Delta y_t$ follows an ARMA(1,3) process.

The additional MA terms in the ARMA processes for P_t and Y_t lead to straightforward modifications for the expressions for variances and covariances.

11. Results for the alternative model

The least squares parameter estimates for the alternative model are presented in Table 4. In all countries the minimized least squares objective function is lower in Table 4 than in Table 2, although the differences are slight. This could reflect the fact that the alternative model has additional MA terms for equilibrium inflation and output growth, although these additional terms are constrained to be functions of the same number of underlying parameters.

TABLE 4. Least squares parameter estimates for the alternative model

Parameter	Germany	France	U.K.	Netherlands	Italy	U.S.
$\tanh^{-1}(\rho)$	1.1955	1.2388	1.5784	1.5621	1.2463	1.1127
$\tanh^{-1}(\psi)$	0.5572	0.6068	-0.0138	0.1615	0.4961	0.1055
γ	4.3078	3.0906	9.1982	3.8273	2.9563	2.9736
Γ	0.7203	0.2368	0.4534	0.7632	0.2877	0.8770
$\tanh^{-1}(F)$	0.0796	1.3356	-2.2358	1.2361	-0.1399	0.2833
σ_{s^P}	0.004598	0.002595	0.001812	0.001851	0.003008	0.00516
σ_{s^T}	0.007858	0.006755	0.061102	0.035266	0.006369	0.01253
σ_{d^P}	0.014554	0.006578	0.043581	0.016609	0.054067	0.01781
σ_{d^T}	0.016308	0.058139	0.000048	0.000002	0.068094	0.000003
LS objective	0.09126	0.13736	0.02689	0.05283	0.08388	0.12788

The weighted least squares estimates are in Table 5. These are below the corresponding minimized values of the weighted least squares objective function in Table 3 only for France and the UK. Nevertheless, the differences are again small.

It is comforting that in many cases, the estimated parameters in Table 5 are quite similar to the ones in Table 3, suggesting that the specification of the length of the information lag does not make a huge difference. The most notable changes are in Italy and the UK, where the alternative specification leads to the expected positive values for F and Φ. By contrast, for Germany the alternative specification leads to counter-intuitive negative values for F and Φ, as in the first column of Table 3. As for France and the Netherlands, the results in Table 5 are preferable to those in Table 3 because various parameters are more tightly estimated in Table 5. Accordingly, in the subsequent discussion, we shall take the second model from Table 3 for Germany but the models from Table 5 for the remaining countries.

TABLE 5. Weighted least squares parameter estimates[a] for the alternative model

Parameter	Germany	France	U.K.	Netherlands	Italy	U.S.
$\tanh^{-1}(\rho)$	1.2022	1.2778	1.5935	1.5071	1.3298	1.1813
	(0.0787)	(0.0975)	(0.1235)	(0.2154)	(0.1096)	(0.0884)
$\tanh^{-1}(\psi)$	0.5247	0.6284	-0.0268	0.1489	0.5202	0.1215
	(0.9572)	(0.0596)	(0.0639)	(0.1391)	(0.1908)	(0.0434)
γ	4.4632	2.7966	10.140	3.6867	3.2231	2.9918
	(3.9632)	(0.7171)	(2.4995)	(0.7617)	(0.4769)	(0.5872)
Γ	0.7525	0.2501	0.4651	0.7935	0.2619	0.8373
	(0.8999)	(0.0294)	(0.0805)	(0.3586)	(0.1468)	(0.0917)
$\tanh^{-1}(F)$	-0.3672	1.2128	0.1955	1.1184	0.1501	0.3126
	(8.3042)	(0.2629)	(1.9226)	(0.6628)	(2.1774)	(0.3179)
σ_{s^P}	0.004418	0.002515	0.001751	0.001872	0.002272	0.004458
	(0.00043)	(0.00029)	(0.00038)	(0.00076)	(0.00055)	(0.00059)
σ_{s^T}	0.007292	0.002264	0.062086	0.033705	0.006335	0.012808
	(1.3160)	(0.00597)	(0.01522)	(0.00540)	(1.4533)	(0.00426)
σ_{d^P}	0.015046	0.011677	0.037835	0.017061	0.050400	0.016556
	(0.00319)	(0.00842)	(0.00718)	(0.00427)	(0.00911)	(0.00194)
σ_{d^T}	0.015608	0.047338	1.5e-06	1.9e-09	0.068144	4.178e-08
	(0.04302)	(0.01529)	(3.7133)	(214.89)	(0.04257)	(39.081)
χ^2 (18 d.f.)	9.450	6.939	8.031	9.907	8.723	9.1907
(P-value)	(0.948)	(0.991)	(0.978)	(0.935)	(0.966)	(0.955)
Implied parameter values[b]						
ρ	0.8343	0.8559	0.9207	0.9064	0.8692	0.8279
ψ	0.4813	0.5570	-0.0268	0.1478	0.4779	0.1209
F	-0.3516	0.8375	0.1930	0.807	0.1490	0.3028
Φ	-0.1957	1.2890	0.1113	3.318	0.0459	0.3636
δ^{-1}	-0.2601	5.1540	0.2392	4.181	0.1751	0.4342
$\beta+\delta^{-1}\alpha^{-1}$	1.329	3.999	2.150	1.260	3.818	

a. Standard errors are in parentheses below each parameter estimate.

b. The income elasticity of money demand β and the real interest elasticity of demand α cannot be recovered.

12. Discussion of the preferred models for each country

For convenience, the parameter estimates in the preferred models for each country are repeated in Table 6. Figures 1–5 graph, for the preferred models for each country, the fit between the sample and the estimated moments, the decomposition of each of the moments into the components arising from each type of shock, and the implied impulse response functions from each type of shock.

The fit between the sample moments and the weighted least squares theoretical moments is presented in the upper left chart of Figures 1–5. The graphs indicate a reasonably close fit between the theoretical and sample moments for all countries. The most difficult problem seemed to be matching the autocovariances of output growth. In all countries, some autocovariances of output growth were positive, while at other lags they were negative. The estimated models generally could not match such patterns well.

TABLE 6. Parameter estimates for the preferred model for each country

Parameter	Germany	France	U. K.	Netherlands	Italy
ρ	0.8301	0.8559	0.9207	0.9064	0.8692
ψ	0.1859	0.5570	-0.0268	0.1478	0.4779
γ	3.6006	2.7966	10.140	3.6867	3.2231
Γ	1.1765	0.2501	0.4651	0.7935	0.2619)
Φ	0.298	1.2890	0.1113	3.318	0.0459
F	0.2021	0.8375	0.1930	0.8070	0.1490
σ_{s^P}	0.004467	0.002515	0.001751	0.001872	0.002272
σ_{s^T}	0.015464	0.002264	0.062086	0.033705	0.006335
σ_{d^P}	0.015676	0.011677	0.037835	0.017061	0.050400
σ_{d^T}	1.4e-08	0.047338	1.5e-06	1.9e-09	0.068144
δ^{-1}	0.253	5.1540	0.2392	4.181	0.1751
$\beta+\delta^{-1}\alpha^{-1}$	0.850	3.999	2.150	1.260	3.818

The most consistent and tightly-estimated parameter is ρ, the autocorrelation coefficient in the aggregate supply curve. Apparently the data are indicating that supply disturbances, whether temporary or permanent, exhibit strong persistence. By contrast, the autocorrelation coefficient in aggregate demand (ψ) implies moderate persistence in France and Italy, weak persistence in Germany and the Netherlands and no persistence at all in the UK.

The estimated elasticities of supply with respect to unexpected inflation, γ, are all of the hypothesized positive sign. The inverse of γ can be interpreted as the slope coefficient in an expectations-augmented Phillips curve. The estimates appear reasonable for all countries except the U.K.

The effect of expected future shocks on current output and prices is determined by $F = \Phi/(\Phi+\Gamma)$. While the estimates of F appear reasonable for all countries in so far as they are all between zero and one, the combined estimates of F and Γ imply unreasonably low interest semi-elasticities of money demand (δ^{-1}) for Germany, the UK and Italy. On the other hand, the estimates of Γ for all countries except Germany can accommodate an income elasticity of money demand (β) of around unity.

The estimated standard deviations of the shocks suggest that temporary demand shocks have been relevant only in France and Italy, where they have been the largest type of shock. Permanent demand shocks appear to have been relatively large in all countries, but particularly so in Italy and the U.K. On the other hand, temporary supply shocks have tended to exceed permanent supply shocks in all countries except France, where the estimated standard errors are similar. The temporary supply shocks are estimated to be substantially larger than the permanent ones in Germany, the UK and the Netherlands. The estimated standard error of permanent supply shocks is more similar across the economies than is the case for any of the other shocks. The similarity in size of permanent supply shocks across economies might suggest that the economies have faced common technology, oil price or other permanent supply shocks.

The relative contributions of the different shocks to variances, autocovariances and cross-covariances in output growth and inflation depend not only on the estimated standard errors of the shocks but also on the autoregressive and moving average coefficients. In the VAR literature, the traditional way to present the information contained in the estimated coefficients is to graph the impulse response functions. Using the parameter estimates in Table 6 we can calculate the effects on Y and P of a unit shock to s_t^P, s_t^T, d_t^P or d_t^T. The resulting impulse response functions for a period of 12 quarters (3 years) are graphed for each country in the final two panels of Figures 1 through 5.

In all countries, permanent supply shocks have the longest lasting effects on output growth, with the peak positive effects occurring after a two or three quarter lag. The effects of the remaining shocks on output growth are negligible beyond two or three quarters after the period of the shock. Permanent supply shocks also have the longest lasting effects on inflation, although permanent demand shocks also have a cumulative positive impact on inflation in all countries.

The *cumulative* effects of shocks on output growth and inflation can also be interpreted as long run effects on the output and price *levels*. From the sums of the impulse responses in Figures 1 through 5, and using the fact that subsequent coefficients will decline exponentially from the final coefficients at lag 12, we can conclude that permanent supply shocks have a long run positive impact on output, and a substantial negative impact on the long run price level, in all countries. The long run effects on output of the remaining shocks are all effectively zero. Furthermore, the long run effects of permanent demand shocks on the price level are very close to unity in all countries.[21]

The middle panels of Figures 1 through 5 graph the contribution of each shock to the variances and autocovariances of Y and P and the contemporaneous and lagged covariances between Y and P. These are not variance decompositions as usually derived and discussed in the VAR literature. Instead of presenting the proportion of forecast error variances resulting from each shock, the figures simply decompose the differ-

21. This is another limitation of the simple supply specification (1)that we plan to address in future work. Even if a permanent demand shock had a non-zero effect on p_t, (1)implies it would have a zero effect on y_t. But then the aggregate demand curve (8) implies the long run effect of d_t^P on p_t must be unity. Buiter (1995, note 13) has argued that the restriction, used by Blanchard and Quah (1989) and others, that demand shocks have no long-run real effects, makes sense for nominal, but not real, demand shocks.

ent variances and covariances into the components coming from each type of shock. In the figures, vertical bars give the contribution by the four types of shock to each variance or covariance. In all cases the bars are ordered, from left to right, as follows: permanent supply shocks, temporary supply shocks, permanent demand shocks, and temporary demand shocks. In some cases (usually involving the temporary shocks) a bar is so small relative to the others that it is not visible on the chart.

In four out of five countries, demand shocks (either permanent or temporary) are the predominant source of variance in output growth. In Germany, the UK and the Netherlands, permanent demand shocks are the largest contributor to variance in output growth. While these shocks are also an important source of output growth variance in Italy, temporary demand shocks are about twice as important. In France, all four shocks contribute a roughly similar amount to the variance of output growth.

The variance of inflation shows no consistent pattern. In Germany and especially Italy, the variance of inflation is predominantly attributable to permanent demand shocks, while in France and the UK, permanent supply shocks are more important. In the Netherlands, temporary supply shocks are the largest source of variance in inflation.

As for the patterns of autocorrelation in output growth and inflation, inspection of the upper-left charts in Figures 1-5 shows that in most cases, inflation is more serially-correlated than output growth. The charts of decompositions show that our model accounts for these patterns by having both permanent supply and demand shocks contributing to positive autocovariances of inflation, whereas in the case of output growth, the two permanent shocks have tended to offset one another, at least at the longer lags. In particular, at the longer lags (and sometimes at all non-zero lags) the permanent supply shocks contribute toward positive autocorrelation of output growth, but permanent demand shocks contribute toward negative autocorrelation.

As discussed earlier, the cross-covariances between output growth and current or future inflation are small and variable (sometimes positive, sometimes negative). By contrast, the cross-covariances between output growth and past inflation are consistently negative and relatively large. Our estimated model largely attributes this difference to a switch in the sign of the effect of permanent demand shocks.[22] In all countries, permanent supply shocks are an important contributor to negative covariance between Y and P at all leads and lags. Permanent demand shocks tend to reinforce the effect of permanent supply shocks in

22. Temporary supply shocks contribute to the negative contemporaneous covariance between output growth and inflation in all countries, but affect other lead or lag covariances only in France, and even then only slightly. In Italy, temporary demand shocks are an important contributor to covariance between output growth and inflation at a number of leads and lags.

the case of covariances between Y and lagged values of P, but they tend to have an offsetting positive effect on covariances between Y and current or future values of P.

We do not present time series of the driving shocks for each country since the sample values of these shocks are not identified. We have four driving shocks, but only two endogenous variables (output and prices). As we remarked in the introduction, an advantage of the method of moments procedure used in this paper is that the number of driving shocks can exceed the number of endogenous variables.

13. Concluding remarks

This paper uses a method of moments procedure to estimate an aggregate demand/aggregate supply model with rational expectations for various European economies. The results indicate that permanent demand shocks are the predominant source of variance in output growth in most of these economies (France is the exception) though permanent supply shocks have important effects on covariance patterns, while the temporary shocks are also significant in France and Italy. Permanent supply shocks are also very significant determinants of the variance and autocorrelation in inflation.

14. Appendix – data sources

The data for this paper were obtained from the International Monetary Fund's International Financial Statistics (IFS) or from the BIS. In the case of West Germany, industrial production was taken from the BIS, series SBBBDE91, and producer prices were also taken from the BIS, series VBBBDE02; data were available from 1962 through 1994. In the cases of the Netherlands and the United Kingdom, data were taken from IFS (line numbers 66..c and 63), and data were available from 1960 through 1994. For Italy, data were taken from IFS and were available from 1960 through the third quarter of 1993. For France, data on industrial production were taken from IFS, while data on producer prices were taken from the BIS, series VBNBFR02; data were available from 1970 through 1994.

There were several reasons for our choice of quarterly data. First, preliminary analysis of the data for our largest economy, Germany, showed that after conversion to logs, first differencing and removal of a trend, the output series contained considerable month-to-month negative autocorrelation. In our view, this month-to-month negative correlation represents the effects of weather, changes in the number of working days per month as we go from year to year, and perhaps measurement error, and not the business cycle phenomena that are our focus. Second, we expect the lags involved in business cycle fluctuations to last more than a year and perhaps several years. However, the computational burden of fitting long lags is increased when monthly data are used and there are many more autocorrelations and cross correlations to be fit. Third, in the case of France, the only consistent data series on producer prices that covered the period we wished to focus on was not available on a monthly basis. And finally, using quarterly data makes it easier to make comparisons with the results in Cooley and Ohanian (1991) and Kydland and Prescott (1990).

As discussed in the text, unit-root tests indicated that the log of the raw series contained a single unit root; accordingly, all the series were first-differenced. To further ensure stationarity, each differenced series was regressed onto a constant, a linear trend, and seasonal dummies,[23] and the residuals from these regressions were used as our measures of output and prices in the manufacturing sector.

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23. Although the series were seasonally adjusted, it is possible that some seasonal effects could remain in our particular samples if the adjustments were applied on a rolling basis. In fact, we did find a few coefficients on seasonal dummy variables marginally statistically significant at conventional levels. Since we are focusing on only the first six lagged covariances we thought it important to ensure all seasonal effects were removed.

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