BARGAINING POWER OF A COALITION IN PARALLEL BARGAINING: ADVANTAGE OF MULTIPLE CABLE SYSTEM OPERATORS

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The paper shows that integrating two players on the same side of two *independent* bilateral monopoly markets can increase their bargaining power. A leading example of such a situation is bargaining between cable operators and broadcasters regarding the carriage of broadcasters' signals on cable systems in two separate markets. From the modeling point of view, one innovation the paper introduces is to generate a coalition's preferences by aggregating the preferences of its members. (*JEL* C78, L41, L96, L98).

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1. We have benefited from the careful reading of an earlier version and insightful comments by Roman Inderst and Christian Wey.

1 . INTRODUCTION

THIS PAPER IS ORIGINALLY MOTIVATED by a topical issue in the television industry. We analyze the issue using a bargaining model that has elements not considered before. The theoretical framework we develop here would be useful in addressing similar issues in other industries as well.

The Cable Television Consumer Protection and Competition Act of 1992 (Cable Act of 1992 hereinafter) allowed a broadcaster to demand compensation from the cable operator that carries the broadcaster's signal. Before this legislation, a cable operator could freely retransmit programs which were initially broadcast over the air.[2]

One interesting issue is whether there are gains from forming coalitions among cable system operators across local markets. Some authors such as Waterman (1996) and Chipty (1994) have argued, without proving it, that multiple cable system operators (or MSOs) have an advantage over unintegrated cable system operators in negotiations with broadcasters. Policy makers also seem to be concerned about the "market power" of integrated cable systems. For instance, the Cable Act of 1992 orders the Federal Communications Commission to establish a reasonable limit on the number of subscribers an MSO can reach.[3] Similar restrictions limit the across-local-market integration of local distributors in other industries, such as movie theater chains.

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2. Broadcasters had been lobbying for this legislation for some time. To their disappointment, however, they received little compensation from cable operators. Chae (1996) analyzes this problem using a bargaining model and provides an explanation for what happened.
3. Congress of the United States (1992), Section 11(c).

It is not clear, however, that MSOs have any advantage. Even though some bargaining models generate gains from forming coalitions in certain environments,[4] there is no bargaining theory we are aware of that explains the advantage of integration across *independent* markets. In this paper, we consider two initially separate local markets and investigate the effect of integration between two players on the same side of the two markets, say the cable operators.

The integrated cable operator or the MSO bargains with the broadcasters in the two markets simultaneously. We adapt the Nash bargaining solution to this "parallel" bargaining problem. In effect, we generalize the Nash solution in two separate directions. First, we generalize it to a situation where one party is a coalition of two players. Second, we generalize it to a situation where one party bargains with opponents on two fronts.

Regarding the generalization of the Nash solution to a situation where one party is a coalition, our approach differs from existing models. Existing models either assume that the coalition's preferences are the same as those of an agent to whom the negotiation is delegated[5] or assume that the coalition's preferences are the same as those of a representative player (assuming that all players in the coalition have the same preferences).[6] By contrast, we assume that the coalition's preferences are aggregated from its members' preferences.

The solution depends on the contract within the coalition of cable operators. We consider two types of the internal contract, one where they can costlessly write a binding contract and the other where no commitment on how to split future payoffs between the members of the coalition is pos-

4. See, for example, Horn and Wolinsky (1988a, 1988b) and Jun (1989).
5. See the literature on strategic delegation referenced, for instance, in Segendorff (1998).
6. See, for example, Jun (1989).

sible. We show, for each type of contract, that the across-market integration is profitable under certain conditions.

There are two intuitive explanations for the results. First, when the integrated party negotiates with each of the other parties, it takes the outcome of the bargaining with the other party as given. This increases the integrated party's fall-back position. To the extent that this makes the integrated party bolder in bargaining, it increases its share. This explanation can be called the *fall-back position effect*. Second, splitting the risk of a breakdown between two members of a coalition can make both of them bolder. This increases the coalition's share. This explanation can be called the *risk-sharing effect*.

If we define *bargaining power* as the relative advantage of a player due to certain characteristics of the player or bargaining environments, we may say that forming a coalition increases bargaining power. If we define *market power* as one's ability to affect market prices to one's advantage, the results of this paper support the view that across-local-market integration increases market power. In our model, this increase in the market power is due to an increase in bargaining power.

In Section 2, we introduce the concept of *risk concession*, based on Zeuthen (1930)'s pioneering work. We then define the Nash solution in terms of marginal risk concessions. Section 3 then extends the framework to the case of an integrated player bargaining in two markets. In Subsection 3.2, we solve this parallel bargaining problem for the case where no-commitment is possible in the within-coalition contract. Then we identify conditions under which the members of the coalition gain from integration. In Subsection 3.3, we solve the parallel bargaining problem for the case where the members of the coalition can write a binding within-coalition contract. Under the additional assumption that agents are risk averse, we show that integration is profitable if certain aggregation conditions are met. Section 4 provides the conclusion.

2 . PRELIMINARIES ON THE BARGAINING SOLUTION

In order to be able to generalize the Nash solution to a bargaining situation involving a coalition, we need to identify the defining characteristic of the solution which is generalizable. In the risk-preference framework, the Nash solution is equivalent to the solution proposed by Zeuthen (1930). The latter is defined as follows: If there are two different positions currently maintained by two negotiating parties, each party has a maximum probability such that the party is willing to risk the probability of a breakdown by insisting on her current position rather than accepting the other party's position. A party whose maximum such probability is not greater than the other's has to make some concession. Thus, the negotiation stops at a single point where the two probabilities are both equal to zero. Even though Nash introduced his solution by certain axioms requiring some desirable properties of the solution in the utility space, it turns out that Zeuthen's solution yields the Nash solution in the utility space if the preferences of the negotiating parties are represented by expected utility functions.[7]

In this paper, we will use Zeuthen's idea to generalize the Nash solution to situations involving a coalition. In a pie-splitting problem, Zeuthen's solution equalizes what we call the "marginal risk concessions" of two players. Thus we will need to define the marginal risk concession of a coalition in order to prescribe a solution for a situation where at least one of the negotiating parties is a coalition of players.

There is another direction in which we need to generalize the Zeuthen-Nash solution in order to be able to analyze a bargaining situation involving a coalition. In certain situations, a coalition

7. This was shown by Harsanyi (1956).

may be able to write an internal contract to divide up the spoil from bargaining with another party. Since this external bargaining can result in an agreement or a breakdown, the internal contract has to specify how the spoil is divided for each contingency. Thus, during the internal bargaining process, the members of a coalition face the problem of bargaining over a contingent pie. We will generalize the Zeuthen-Nash solution to this contingent-pie problem by requiring that the players optimally share risks across different states of nature.

The necessary generalizations will be done in the next section. In this section, we will briefly (but carefully) look at a standard two-person bargaining problem to introduce our framework, terminology, and notation, which we will use in the next section.

2.1. *Preferences over Lotteries*

A lottery $l\colon \boldsymbol{R}_+ \to [0, 1]$, where $\boldsymbol{R}_+$ is the set of nonnegative real numbers, is a discrete probability function: there exist $x_1, \ldots, x_n \in \boldsymbol{R}_+$ such that $l(x_1) + \ldots + l(x_n) = 1$ and $l(x) = 0$ if $x \notin \{x_1, \ldots, x_n\}$. The lottery space, denoted $L(\boldsymbol{R}_+)$, is the set of all lotteries equipped with the following operation: for any $l, m \in L(\boldsymbol{R}_+)$ and $p \in [0, 1]$, the lottery

$p \bullet l \oplus (1-p) \bullet m : \boldsymbol{R}_+ \to [0, 1]$ is defined by

$$(p \bullet l \oplus (1-p) \bullet m)(x) = p \cdot l(x) + (1-p) \cdot m(x) \text{ for any } x \in \boldsymbol{R}_+ .$$

As is well known, the lottery space is a convex linear space, that is, satisfies the following properties[8]:

L1. $1 \bullet l \oplus 0 \bullet m = l$

8. See Herstein and Milnor (1953).

L2. $p \bullet l \oplus (1-p) \bullet m = (1-p) \bullet m \oplus p \bullet l$
L3. $q \bullet (p \bullet l \oplus (1-p) \bullet m) \oplus (1-q) \bullet m = (qp) \bullet l \oplus (1-qp) \bullet m$

We will identify a number $x \in \boldsymbol{R}_+$ with a sure lottery $\tilde{x} \in L(\boldsymbol{R}_+)$ such that $\tilde{x}(x) = 1$.[9] A player has a complete and transitive preference relation $\precsim$ on the lottery space that satisfies the following three axioms:

ASSUMPTION 1: *(Smoothness) If $m \succsim l \succsim n$, where $m \succ n$, there exists a unique number $h(l, m, n) \in [0, 1]$ such that*

(i) $l \sim h(l, m, n) \bullet m \oplus \{1 - h(l, m, n)\} \bullet n$,

(ii) Let $\hat{h}(x, m, n) = h(\tilde{x}, m, n)$ for $x \in \boldsymbol{R}_+$. Then $\hat{h}(x, m, n)$ is a smooth function of x such that $\dfrac{\partial \hat{h}}{\partial x}(x, m, n) > 0$.

ASSUMPTION 2: *(Independence) If $l \sim l'$, then for any m and any $p \in [0, 1]$,* $p \bullet l \oplus (1-p) \bullet m \sim p \bullet l' \oplus (1-p) \bullet m$.

ASSUMPTION 3: *(Monotonicity) If $x > y$ (where $x, y \in \boldsymbol{R}_+$), then $\tilde{x} \succ \tilde{y}$.*

It is well known that an expected utility function exists under the assumptions of continuity and independence. Replacing continuity with smoothness yields a stronger set of axioms, and thus an expected utility function exists under our assumptions. We introduce the smoothness assump-

9. We will use the notation $\tilde{x}$ only if it is necessary to make the conceptual distinction between x and $\tilde{x}$.

tion because we need it to define the concept of marginal risk concession.[10] *A la* Herstein and Milnor (1953), we can represent a player's preferences by a utility function.

PROPOSITION 2.1: *There exists a unique function* $V: L(\boldsymbol{R}_+) \to \boldsymbol{R}$ *that satisfies* $V(0) = 0, V(1) = 1$*, and*

(i) $l \succ m$ *if and only if* $V(l) > V(m)$,

(ii) $V(p \bullet l \oplus (1-p) \bullet m) = pV(l) + (1-p)V(m)$,

(iii) Put $v(x) = V(\tilde{x})$ *for* $x \in \boldsymbol{R}_+$*. Then* $v(x)$ *is a smooth function of x such that* $v'(x) > 0$ *for* $x > 0$.

The proof of the proposition is similar to Herstein and Milnor's (1953) and thus will be omitted here. One may call the function *V* the von Neumann-Morgenstern utility function over lotteries and the function *v* the von Neumann-Morgenstern utility function over prizes. The following proposition is obvious:

PROPOSITION 2.2: *If* $l \precsim 1$ *then* $V(l) = h(l, 1, 0)$*, and if* $l \succ 1$ *then* $V(l) = 1/h(1, l, 0)$.

In order to understand the concept of risk concession, which will be introduced in the next subsection, it is necessary to study the certainty equivalent of a lottery.

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10. A smooth function is one that is differentiable as many times as one wants. For the results of this paper, it is sufficient that the function $\hat{h}(x, m, n)$ is three times differentiable with respect to x.

DEFINITION 2.1: The *certainty equivalent* of a lottery $p \bullet y \oplus (1-p) \bullet z$ is a sure payoff $s(p, y, z) \in \boldsymbol{R}_+$ that satisfies $s(p, y, z) \sim p \bullet y \oplus (1-p) \bullet z$.

PROPOSITION 2.3: *Let y* $\succ$ *z. Then* $s(p, y, z)$ *is a smooth function of p such that* $\frac{\partial s}{\partial p}(p, y, z) > 0$.

PROOF: $x = s(p, y, z)$ is a smooth function of *p* such that $\frac{\partial s}{\partial p}(p, y, z) > 0$ because it is the inverse function of $p = \hat{h}(x, y, z)$, which is a smooth function of *x* such that $\frac{\partial \hat{h}}{\partial x}(x, y, z) > 0$.

Q.E.D.

PROPOSITION 2.4: *Let y > z. If* $y \geq x \geq z$, *one has*

$$x = s\left(\frac{v(x) - v(z)}{v(y) - v(z)}, y, z\right).$$

PROOF: By Assumption 1, there exists some *p* such that

$$x \sim p \bullet y \oplus (1-p) \bullet z.$$

We have only to show that $p = \frac{v(x) - v(z)}{v(y) - v(z)}$. But this follows from $v(x) = pv(y) + (1-p)v(z)$.

Q.E.D.

PROPOSITION 2.5: *Let y > z. Then* $\frac{\partial s}{\partial p}(1, y, z) = \frac{v(y) - v(z)}{v'(y)}$.

PROOF: Differentiating the expression in Proposition 2.4 with respect to *x*, one obtains

$$1 = \frac{\partial s}{\partial p}\left(\frac{v(x) - v(z)}{v(y) - v(z)}, y, z\right) \cdot \frac{v'(x)}{v(y) - v(z)}.$$

Setting $x = y$ yields

$$1 = \frac{\partial s}{\partial p}(1, y, z) \cdot \frac{v'(y)}{v(y) - v(z)},$$

from which the desired equality follows. *Q.E.D.*

2.2. *Two-Person Bargaining Problem*

DEFINITION 2.2: A *bargaining problem* $\langle (i, j), \pi, (d_i, d_j) \rangle$, where $d_i, d_j \geq 0$ and $\pi > d_i + d_j$, is a situation where two players (i, j) split a pie of size π if they can agree on their shares, and receive the breakdown payoffs (d_i, d_j) otherwise.

In order to introduce the solution to the bargaining problem, we first need to focus on some properties of preferences. For simplicity, we will drop the subscripts for players until we need them.

During the process of bargaining, a player typically faces a gamble $p \bullet (x + d) \oplus (1 - p) \bullet d$, where $x + d$ ($\geq d$) is her payoff in the event of an agreement, d (≥ 0) is her payoff in the event of a breakdown, and $1 - p$ the breakdown probability.[11] We will denote such a gamble simply by $(p, x + d, d)$.

DEFINITION 2.3: The *risk concession* of a player facing a gamble $(p, x+d, d)$ is the amount the player is willing to pay to avoid the chance of a breakdown. It will be denoted and defined as $c(p, x+d, d) = x + d - s(p, x+d, d)$.

DEFINITION 2.4: The *marginal risk concession* of a player facing a pair of payoffs $(x+d, d)$ is the rate of change in risk concession as the breakdown probability approaches zero:

$$\lim_{p \to 1} \frac{c(p, x+d, d)}{(1-p)}.$$

It will be denoted $\mu(x+d, d)$.

PROPOSITION 2.6: *One has* $\mu(x+d, d) = \dfrac{v(x+d) - v(d)}{v'(x+d)}$.

PROOF: By Definitions 2.3 and 2.4,

$$\mu(x+d, d) = \lim_{p \to 1} \frac{x + d - s(p, x+d, d)}{(1-p)}.$$

Using L'Hopital's rule, we get

$$\mu(x+d, d) = \frac{\partial s}{\partial p}(1, x+d, d).$$

By Proposition 2.5, $\dfrac{\partial s}{\partial p}(1, x+d, d) = \dfrac{v(x+d) - v(d)}{v'(x+d)}$. *Q.E.D.*

11.Throughout this paper, we will use the term "gamble" for a lottery which is a probability mix of an agreement payoff and a breakdown payoff.

Note that $\mu(x+d, d)$ is a smooth function of x and d. In addition to Assumptions 1-3, we make the following assumption *throughout this paper*:

ASSUMPTION 4: $\mu(x+d, d)$ *is increasing in x for all x > 0.*

Assumption 4 holds for a very general class of preferences. The class includes all preferences exhibiting risk aversion or risk neutrality. It also includes preferences that can be represented by utility functions with constant relative risk aversion.

PROPOSITION 2.7: *The marginal risk concession* $\mu(x+d, d)$ *is increasing in* $x > 0$ *if and only if* $\frac{d}{dx}\log(v(x+d) - v(d))$ *decreases in* $x > 0$.

PROOF: For $x > 0$, one has

$$\frac{d}{dx}\log(v(x+d) - v(d)) = \frac{v'(x+d)}{v(x+d) - v(d)} = \frac{1}{\mu(x+d, d)},$$

from which follows the desired result. *Q.E.D.*

That Assumption 4 holds for all risk averse or risk neutral preferences, that is, those with $v''(\ ) \le 0$, can be easily seen from Proposition 2.7, for

$$\frac{d^2}{dx^2}\log(v(x+d) - v(d)) = \frac{v''(x+d)[v(x+d) - v(d)] - [v'(x+d)]^2}{[v(x+d) - v(d)]^2}.$$

That Assumption 4 is also satisfied by all utility functions v with constant relative risk aversion is shown in Appendix A. In particular, the concavity of the function $v(x)$ is not a necessary condition for Assumption 4.

We will now define the Nash bargaining solution in terms of players' marginal risk concessions and state two properties of the Nash solution that will be used in Section 3.

DEFINITION 2.5: The *Nash solution* of a bargaining problem $\langle (i, j), \pi, (d_i, d_j) \rangle$ is a vector $(x_i + d_i, x_j + d_j)$ such that $x_i + d_i + x_j + d_j = \pi$ and

$$\mu_i(x_i + d_i, d_i) = \mu_j(x_j + d_j, d_j) .$$

The Nash solution will be denoted

$$N\langle (i, j), \pi, (d_i, d_j) \rangle = (N_i\langle (i, j), \pi, (d_i, d_j) \rangle, N_j\langle (i, j), \pi, (d_i, d_j) \rangle) .$$

PROPOSITION 2.8: *There exists a unique Nash solution to the bargaining problem* $\langle (i, j), \pi, (d_i, d_j) \rangle$.

PROOF: The Nash solution satisfies the following equation

$$\mu_i(x_i + d_i, d_i) = \mu_j(\pi - x_i - d_i, d_j) .$$

If one sets $x_i = 0$, the left hand side of the above equation is zero while the right hand side is positive. If one sets $x_i = \pi - d_i - d_j$, the left hand side of the above equation is positive and the right hand side is equal to zero. Since, by Proposition 2.6 and Assumption 4, the left hand side is continuously increasing in x_i while the right hand side is continuously decreasing in x_i, there exists a unique solution. *Q.E.D.*

PROPOSITION 2.9: $N_i\langle(i, j), \pi, (d_i, d_j)\rangle$ *is an increasing and smooth function of* π *for* $i = 1, 2$.

PROOF: Follows from the proof of Proposition 2.8. *Q.E.D.*

In the time-preference framework, Chae (1993) defines the Nash solution as a payoff vector equalizing "marginal impatience" among all players and establishes propositions analogous to the above two propositions. The mathematical structure of the proofs of the above two propositions is essentially the same as that of the corresponding propositions in Chae (1993).

2.3. *Bargaining over a Contingent Pie*

In Subsection 3.3, we need to deal with a bargaining situation where players bargain over a contingent pie whose size depends on the realized state of nature. Thus in this subsection, we will extend the analysis of the previous subsection to cover such a situation. For the analyses of Subsection 3.3, we will assume that players are risk averse, that is, they prefer the expected value of a gamble to the gamble itself. Thus we will make the same assumption in this subsection.

Suppose that there are two states of nature, σ and τ, which occur with probabilities q and $1 - q$, respectively. Two players have to agree on how to split the pie π^s in each state $s \in \{\sigma, \tau\}$ in order to avoid the chance of a breakdown. The contingent pie $\pi = (\pi^\sigma, \pi^\tau)$ is equivalent to the lottery $q \bullet \pi^\sigma \oplus (1 - q) \bullet \pi^\tau$, where we assume that there exists some division of $\pi = (\pi^\sigma, \pi^\tau)$ that both players prefer to their break-down payoffs. We define the bargaining problem over this contingent pie as follows:

DEFINITION 2.6: A *contingent-pie bargaining problem* $\langle (i, j), \pi, (d_i, d_j) \rangle$, where $d_i, d_j \in \boldsymbol{R}_+$ and there exist some contingent payoffs y_i, y_j such that $y_i \succ_i d_i$, $y_j \succ_j d_j$, and $y_i + y_j = \pi$, is a situation where two players have to agree on how to split a contingent pie π in order to avoid a breakdown.

Note that the breakdown position of each player is a non-contingent payoff. Without much loss of generality, we assume that players bargain over Pareto efficient splits of the contingent pie. That is, we require that in each state of nature the entire pie is split between the two players and that players share risks optimally across different states of nature. When players are risk averse, this entails that the marginal rates of substitution between different states of nature, as formally defined below, are equalized across players.

For any $x, y \in \boldsymbol{R}_+$ such that $x > y > 0$, define $\xi_q(\delta)$ for sufficiently small $|\delta|$ by the following indifference relation:

$$q \bullet x \oplus (1-q) \bullet y \sim q \bullet (x - \delta) \oplus (1-q) \bullet (y + \xi_q(\delta)).$$

DEFINITION 2.7: The *marginal rate of substitution for a fair gamble* between x and y is denoted and defined by $m(x, y) = \xi'_{\frac{1}{2}}(0)$.

PROPOSITION 2.10: $m(x, y) = \dfrac{v'(x)}{v'(y)}$.

PROOF: From Definition 2.7 and the equality

$$\xi'_q(0) = \frac{q}{(1-q)} \cdot \frac{v'(x)}{v'(y)},$$

follows the result. *Q.E.D.*

Under the assumption of risk aversion, one can denote and characterize the set of Pareto efficient splits of the contingent pie $\pi = (\pi^\sigma, \pi^\tau)$ as

$$PE = \left\{(y_i^\sigma, y_i^\tau);\ \frac{q}{1-q} m_i(y_i^\sigma, y_i^\tau) = \frac{q}{1-q} m_j(\pi^\sigma - y_i^\sigma, \pi^\tau - y_i^\tau)\right\}$$

$$= \left\{(y_i^\sigma, y_i^\tau);\ m_i(y_i^\sigma, y_i^\tau) = m_j(\pi^\sigma - y_i^\sigma, \pi^\tau - y_i^\tau)\right\}$$

using player i ‘s contingent payoff to denote the split of the contingent pie. The set is a one-dimensional manifold, that is, a smooth curve. Since the bargaining will break down if either player is not given a contingent payoff that will make her at least as well off as at the breakdown point, the relevant part of PE is the *core*

$$C = \{y_i \in PE;\ y_i \succsim_i d_i \text{ and } \pi - y_i \succsim_j d_j\}.$$

In the Edgeworth Box of Figure 1, PE is the solid curve from the south-west corner to the north-east corner, and C is the thick part.

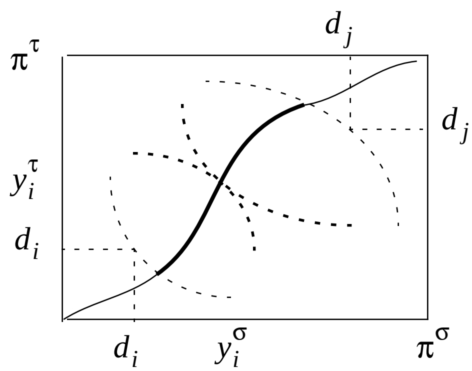

FIGURE 1

PROPOSITION 2.11: *Suppose* $y_i, \hat{y}_i \in C$. *If* $y_i^\sigma > \hat{y}_i^\sigma$ *then* $y_i^\tau > \hat{y}_i^\tau$.

PROOF: Assume otherwise, that is $y_i^\sigma > \hat{y}_i^\sigma$ and $y_i^\tau \le \hat{y}_i^\tau$. Then risk aversion implies

$$\frac{v_i'(y_i^\sigma)}{v_i'(y_i^\tau)} < \frac{v_i'(\hat{y}_i^\sigma)}{v_i'(\hat{y}_i^\tau)} \text{ and } \frac{v_j'(\pi^\sigma - y_i^\sigma)}{v_j'(\pi^\tau - y_i^\tau)} > \frac{v_j'(\pi^\sigma - \hat{y}_i^\sigma)}{v_j'(\pi^\tau - \hat{y}_i^\tau)},$$

which is impossible because both y_i and $\hat{y}_i$ are Pareto efficient. *Q.E.D.*

During the process of bargaining over a contingent pie $\pi = (\pi^\sigma, \pi^\tau)$, a player typically faces a gamble $p \bullet y_i \oplus (1-p) \bullet d_i$, where $y_i = (x_i^\sigma + d_i, x_i^\tau + d_i)$ is her contingent payoff in the event of an agreement, d_i (≥ 0) is her payoff in the event of a breakdown, and $1-p$ the breakdown probability. For simplicity, we will denote such a gamble by (p, y_i, d_i). Note here that the contingent payoff y_i is equivalent to the lottery $q \bullet (x_i^\sigma + d_i) \oplus (1-q) \bullet (x_i^\tau + d_i)$.

As was the case in the bargaining problem over a non-contingent pie, the extent to which a player is willing to concede in order to avoid the chance of a breakdown plays an important role in finding the solution for a bargaining problem over a contingent pie. In order to formalize this notion, we have to first introduce the analogue of certainty equivalent.

DEFINITION 2.8: For any gamble (p, y_i, d_i) where $y_i = (x_i^\sigma + d_i, x_i^\tau + d_i) \in C$, the breakdown-free contingent payoff is defined as a contingent lottery $s_i(p, y_i, d_i) = (s_i^\sigma(p, y_i, d_i), s_i^\tau(p, y_i, d_i)) \in C$ that satisfies

$$s_i(p, y_i, d_i) \sim p \bullet y_i \oplus (1-p) \bullet d_i,$$

i.e.,

$$q \bullet s_i^\sigma(p, y_i, d_i) \oplus (1-q) \bullet s_i^\tau(p, y_i, d_i) \sim$$
$$p \bullet \{q \bullet (x_i^\sigma + d_i) \oplus (1-q) \bullet (x_i^\tau + d_i)\} \oplus (1-p) \bullet d_i.$$

DEFINITION 2.9: The *risk concession* of a player facing a gamble (p, y_i, d_i), where $y_i \in C$, is the amount, along the core, of contingent payoff the player is willing to pay to avoid the chance of a breakdown. It will be denoted and defined as

$$c_i(p, y_i, d_i) = y_i - s_i(p, y_i, d_i).$$

DEFINITION 2.10: The *marginal risk concession* along the core, of a player facing a pair (y_i, d_i), where $y_i \in C$, is the rate of change of the risk concession as the breakdown probability approaches zero:

$$\lim_{p \to 1} \frac{c_i(p, y_i, d_i)}{(1-p)} = \left(\lim_{p \to 1} \frac{c_i^\sigma(p, y_i, d_i)}{(1-p)}, \lim_{p \to 1} \frac{c_i^\tau(p, y_i, d_i)}{(1-p)} \right).$$

It will be denoted $\mu_i(y_i, d_i) = (\mu_i^\sigma(y_i, d_i), \mu_i^\tau(y_i, d_i))$.

The marginal risk concession is well defined because, under the assumption of risk aversion, C is a smooth curve. In Appendix B, we derive the following:

PROPOSITION 2.12: *One has*

$$\mu_i^\sigma(y_i, d_i) = \frac{qv_i(y_i^\sigma) + (1-q)v_i(y_i^\tau) - v_i(d_i)}{v_i'(y_i^\sigma)\left\{q + (1-q)\frac{v_i'(y_i^\tau)}{v_i'(y_i^\sigma)} \cdot \frac{dy_i^\tau}{dy_i^\sigma}\right\}},$$

$$\mu_i^\tau(y_i, d_i) = \frac{qv_i(y_i^\sigma) + (1-q)v_i(y_i^\tau) - v_i(d_i)}{v_i'(y_i^\tau)\left\{q\frac{v_i'(y_i^\sigma)}{v_i'(y_i^\tau)} \cdot \frac{dy_i^\sigma}{dy_i^\tau} + (1-q)\right\}},$$

where dy_i^τ / dy_i^σ is the slope (and dy_i^σ / dy_i^τ its inverse) of the C curve at (y_i^σ, y_i^τ).

Now we can introduce the Nash solution for a contingent-pie bargaining problem.

DEFINITION 2.11: The *Nash solution to a contingent-pie bargaining problem* $\langle (i, j), \pi, (d_i, d_j) \rangle$ is a vector $(y_i, \pi - y_i)$, where $y_i \in C$, that satisfies the following equation:

$$\mu_i(y_i, d_i) = \mu_j(\pi - y_i, d_j) .$$

To prove existence and uniqueness of the Nash solution for a contingent-pie bargaining problem, the following definition is useful:

DEFINITION 2.12: *The marginal risk concession in terms of sure payoff in state* σ of a player facing a pair (y_i, d_i) is defined and denoted as

$$\hat{\mu}_i^{\sigma}(y_i, d_i) = \frac{q v_i(y_i^{\sigma}) + (1-q) v_i(y_i^{\tau}) - v_i(d_i)}{v_i'(y_i^{\sigma})} .$$

$\hat{\mu}_i^{\sigma}(y_i, d_i)$ measures the rate of change of the amount, measured in terms of sure payoff in state σ, player *i* is willing to give up in order to avoid the chance of a breakdown as the breakdown probability approaches zero. In Appendix C, we show the following:

PROPOSITION 2.13: $\mu_i(y_i, d_i) = \mu_j(\pi - y_i, d_j)$ if and only if

$$\hat{\mu}_i^{\sigma}(y_i, d_i) = \hat{\mu}_j^{\sigma}(\pi - y_i, d_j) .$$

Appendix D derives the following proposition:

PROPOSITION 2.14: $\hat{\mu}_i^{\sigma}(y_i, d_i)$ *is increasing in* y_i *along the C curve.*

PROPOSITION 2.15: *If both players are risk averse, there exists a unique Nash solution for the contingent-pie bargaining problem* $\langle (i, j), \pi, (d_i, d_j) \rangle$.

The proof, shown in Appendix E, is similar to that of Proposition 2.8.

In the non-contingent pie case, the Nash solution is often motivated as the limit of the Rubinstein solution for a strategic bargaining model.[12] Since we introduced the contingent-pie bargaining problem in this subsection, it is perhaps our duty to provide a similar motivation for the Nash solution for this case. In Appendix F, we will introduce the Rubinstein solution for our contingent-pie bargaining problem and show that the Rubinstein solution equalizes the risk concessions of the two players. Since the Nash solution equalizes the *marginal* risk concessions of the two players, one can see easily that the Nash solution is the limit of the Rubinstein solution as the breakdown probability goes to zero.

12. Rubinstein's alternating-offer model in the time-preference framework can be converted to a similar model where after every offer there is an exogenous probability that the game ends. This setup replaces the time cost of rejecting an offer by the risk that the game may terminate. See Binmore, Rubinstein, and Wolinsky (1986).

In Appendix G, we relate the above definition of the Nash solution for a contingent-pie bargaining problem to the standard definition representing players' preferences by von Neumann-Morgenstern utility functions as in Nash (1950).

3 . PARALLEL BARGAINING

In this section, we will investigate the consequences of integrating one type of players across different markets. Consider two separate bilateral monopoly markets *A* and *B*. As a leading example, we will consider markets where broadcasters and cable operators negotiate over the terms of carrying broadcast channels on cable systems. In market *A*, cable TV operator *a* and broadcaster $\hat{a}$ bargain over the split of π^A, their net gain from carrying the broadcast channel on the cable system. In market *B*, cable TV operator *b* and broadcaster $\hat{b}$ bargain over the split of their surplus π^B. In the event of a breakdown of bargaining, the profit position of player *i* (= a, $\hat{a}$, b, $\hat{b}$) is d_i. Formally, we have two parallel bargaining problems $\langle (a, \hat{a}), \pi^A, (d_a, d_{\hat{a}}) \rangle$ and $\langle (b, \hat{b}), \pi^B, (d_b, d_{\hat{b}}) \rangle$.

In the benchmark case where players in markets *A* and *B* are independent firms, we posit that the solutions to the bargaining problems in markets *A* and *B* are the Nash solutions $N\langle (a, \hat{a}), \pi^A, (0, 0) \rangle$ and $N\langle (b, \hat{b}), \pi^B, (0, 0) \rangle$, where we have, without loss of generality, normalized the initial fall back positions to be zero.

In what follows, we will investigate how the integration of cable operators across the two markets affects their and the broadcasters' payoffs. The integration pits the coalition of cable operators against the broadcasters of markets *A* and *B* as illustrated in Table I.

TABLE I

	before integration	*after integration*
market A	$a \leftrightarrow \hat{a}$	$\{a, b\} \leftrightarrow \hat{a}$
market B	$b \leftrightarrow \hat{b}$	$\{a, b\} \leftrightarrow \hat{b}$

In order to analyze bargaining between a coalition and its opponents on two fronts, we need to modify the above solution in two different directions. First, we need to specify how the bargaining in one market affects the bargaining in another market. In this regard, we imagine a situation where the two bargaining problems are settled simultaneously rather than sequentially and assume that when players bargain in one market, they take the outcome of bargaining in the other market as given. Second, we need to extend the definition of the Nash solution to a bargaining problem between a coalition and a player. Since the Nash solution is one where the marginal risk concessions of two players are equalized, we will have to define the marginal risk concession of a coalition. This will be defined essentially as the sum of the marginal risk concessions of the two members of the coalition. This makes sense because the risk concession of the coalition measures how much the coalition is willing to give up to avoid the chance of a breakdown, and the amount the coalition is willing to concede will be quite naturally the sum of the amounts the members of the coalition are willing to concede. We emphasize here that our notion of risk concession is a natural extension of Zeuthen's idea.

3.1. *Simultaneous Nash Solution*

We will denote the coalition of cable operators $\{a, b\}$ simply by c. If the bargaining between the coalition and an opponent breaks down, the coalition receives a payoff d_c. If the bargaining ends in an agreement, the coalition receives a payoff $x_c + d_c$.

In general, the marginal risk concession of a coalition will be defined as the sum of the marginal risk concessions of the two members of the coalition. In order to measure the marginal risk concession of each member of the coalition, however, one needs to know how both $x_c + d_c$ and d_c are split between a and b.

Regarding the mechanism to divide a given pie between the two members of the coalition, we will consider two alternative scenarios. In the first scenario, we assume that the cable operators, when they are contemplating whether to form a coalition, cannot commit themselves to any division of the coalition's share of the pie. In this scenario, we are assuming in effect that it is either impossible or prohibitively costly to write a binding contract between the cable operators. In the second scenario, we assume that the cable operators, when they are contemplating whether to form a coalition, can make a binding agreement on how to split the coalition's share.

Denote the division scheme under either scenario by $S(x_c + d_c, d_c)$. The scheme has to specify the shares of a and b in both the agreement and breakdown states. Denote the agreement and breakdown states by σ and τ, respectively. Then

$$S(x_c + d_c, d_c) = (S_a(x_c + d_c, d_c), S_b(x_c + d_c, d_c))$$

where

$$S_i(x_c + d_c, d_c) = (S_i^{\sigma}(x_c + d_c, d_c), S_i^{\tau}(x_c + d_c, d_c)) \text{ for } i = a, b.$$

The precise form of the division scheme $S(x_c + d_c, d_c)$ under each scenario will be introduced in Subsections 3.2 and 3.3.

DEFINITION 3.1: Given a division scheme $S(x_c + d_c, d_c)$, the *marginal risk concession of a coalition* is defined as

$$\mu_c(x_c + d_c, d_c) = \mu_a(S_a^{\sigma}(x_c + d_c, d_c), S_a^{\tau}(x_c + d_c, d_c)) + \mu_b(S_b^{\sigma}(x_c + d_c, d_c), S_b^{\tau}(x_c + d_c, d_c)).$$

Denote the coalition's shares in markets *A* and *B* by x_c^A and x_c^B, respectively. Then in market *A*, one has $(x_c + d_c, d_c) = (x_c^A + x_c^B, x_c^B)$, for the coalition takes x_c^B as given. Similarly, in market *B*, one has $(x_c + d_c, d_c) = (x_c^B + x_c^A, x_c^A)$.

DEFINITION 3.2: A *simultaneous Nash solution* to the parallel bargaining problem with a one-sided coalition is a vector $(x_c^A, x_c^B, x_{\hat{a}}, x_{\hat{b}})$ that satisfies the following equations:

(1) $\qquad \mu_c(x_c^A + x_c^B, x_c^B) = \mu_{\hat{a}}(x_{\hat{a}}, 0)$,

(2) $\qquad \mu_c(x_c^B + x_c^A, x_c^A) = \mu_{\hat{b}}(x_{\hat{b}}, 0)$,

(3) $\qquad x_c^A + x_{\hat{a}} = \pi^A$,

(4) $$x_c^B + x_{\hat{b}} = \pi^B.$$

In the absence of a coalition, the payoffs of cable operators *a* and *b* are $N_a\langle(a, \hat{a}), \pi^A, (0, 0)\rangle$ and $N_b\langle(b, \hat{b}), \pi^B, (0, 0)\rangle$, respectively. To simplify the notation, let

$$n_a = N_a\langle(a, \hat{a}), \pi^A, (0, 0)\rangle,$$
$$n_b = N_b\langle(b, \hat{b}), \pi^B, (0, 0)\rangle.$$

The coalition will actually form only if each member of the coalition gains from joining the coalition. Thus one may consider a stronger solution to the parallel bargaining problem.

DEFINITION 3.3: A *bona fide solution* to the parallel bargaining problem with a one-sided coalition is a simultaneous Nash solution $(x_c^A, x_c^B, x_{\hat{a}}, x_{\hat{b}})$ where each member of the coalition gains from joining the coalition, i.e., $S_i^\sigma(x_c^A + x_c^B, x_c^B) > n_i$ for $i = a, b$.

We will now consider some desirable properties of $\mu_c(x_c + d_c, d_c)$ that may or may not hold in particular environments as will be shown in the next two subsections.

CONDITION 1: $\mu_c(d_c, d_c) = 0$ *for* $d_c \geq 0$.

CONDITION 2: $\mu_c(x_c + d_c, d_c)$ *is an increasing and smooth function of* x_c.

LEMMA 3.1: *If Conditions 1 and 2 are satisfied, there exists a simultaneous Nash solution to the parallel bargaining problem with a one-sided coalition.*

PROOF: Taking x_c^B as given, equations (1) and (3) of Definition 3.2 define the Nash solution to the bargaining problem in market *A*. Substituting (3) into (1), one has

$$\mu_c(x_c^A + x_c^B, x_c^B) = \mu_{\hat{a}}(\pi^A - x_c^A, 0)\,. \tag{5}$$

By Conditions 1 and 2, as x_c^A increases from 0 to π^A, the left hand side of (5) increases from 0 to a positive number while the right hand side decreases from a positive number to 0. Thus there exists a unique solution to (5). That is, for a given breakdown point x_c^B, this bargaining problem has a unique solution, which determines the payoff for the coalition in market *A*, x_c^A. We can thus define an implicit function $x_c^A = \tilde{x}_c^A(x_c^B)$. Since $\mu_c(\ ,\ )$ and $\mu_{\hat{a}}(\ ,\ )$ are smooth, $x_c^A = \tilde{x}_c^A(x_c^B)$ is smooth and thus continuous in particular.

Symmetrically, using equations (2) and (4), we can define a continuous function $\tilde{x}_c^B(x_c^A)$. Thus we have a continuous mapping $(\tilde{x}_c^A(x_c^B), \tilde{x}_c^A(x_c^B))$ from $[0, \pi^A] \times [0, \pi^B]$ to itself. Therefore, there exists a fixed point by Brower's fixed point theorem. *Q.E.D.*

CONDITION 3: *The marginal risk concession of the coalition* $\mu_c(x_c + d_c, d_c)$ *is non-increasing in* d_c.

LEMMA 3.2: *If Conditions 1, 2, and 3 are satisfied, the functions* $\tilde{x}_c^A(x_c^B)$ *and* $\tilde{x}_c^B(x_c^A)$ *in the proof of Lemma 3.1 are smooth and non-decreasing.*

PROOF: Functions $\tilde{x}_c^A(x_c^B)$ *and* $\tilde{x}_c^B(x_c^A)$ are well defined and smooth by Conditions 1 and 2 as shown in the proof of Lemma 3.1. Condition 3 guarantees that the functions $\tilde{x}_c^A(x_c^B)$ and $\tilde{x}_c^B(x_c^A)$ are non-decreasing as can be seen from equation (5) in the proof of Lemma 3.1. *Q.E.D.*

3.2. *No-Commitment Solution*

Consider the case where the cable operators can make no commitment as to the division of the coalition's share of the pie. In this case, they bargain over the division of the total payoff the coalition receives after either an agreement is reached or the bargaining ends in a breakdown.

DEFINITION 3.4: In the case where the members of a coalition can make no commitment as to the division of the coalition's share of the pie, the *division scheme* is defined as

$$(S_i^{\sigma}(x_c + d_c, d_c), S_i^{\tau}(x_c + d_c, d_c)) = (N_i\langle(a, b), x_c + d_c, (0, 0)\rangle, N_i\langle(a, b), d_c, (0, 0)\rangle)$$

for $i = a, b$.

Notice that the share each member of the coalition receives in each of the two states is determined through Nash bargaining inside the coalition. This feature is due to the assumption of no commitment. If the solution is different from the bargaining solution, one member of the coalition will have an incentive to renegotiate. One can combine Definitions 3.1 and 3.4.

PROPOSITION 3.1: *In the case where the members of a coalition can make no commitment as to the division of the coalition's share of the pie, the marginal risk concession can be written as*

$$\mu_c(x_c + d_c, d_c) = \sum_{i = a, b} \mu_i(N_i\langle(a, b), x_c + d_c, (0, 0)\rangle, N_i\langle(a, b), d_c, (0, 0)\rangle).$$

PROPOSITION 3.2: *If no commitment is possible regarding the division of the coalition's share of the pie, Conditions 1 and 2 are satisfied.*

PROOF: That Condition 1 is satisfied is obvious from Definitions Proposition 3.1. By Proposition 2.9, $N_i\langle(a, b), x_c + d_c, (0, 0)\rangle$ is an increasing and smooth function of x_c. Since the function $\mu_i(\ , \ )$ is an increasing and smooth function of its first argument, $\mu_i(N_i\langle(a, b), x_c + d_c, (0, 0)\rangle, N_i\langle(a, b), d_c, (0, 0)\rangle)$ is an increasing and smooth function of x_c for each $i = a, b$. This in turn implies that $\mu_c(x_c + d_c, d_c)$, which is the sum of $\mu_a(.)$ and $\mu_b(.)$, an increasing and smooth function of x_c. *Q.E.D.*

THEOREM 3.1: *If no commitment is possible regarding the division of the coalition's share of the pie, there exists a simultaneous Nash solution to the parallel bargaining problem with a one-sided coalition.*

PROOF: Follows from Lemma 3.1 and Proposition 3.2. *Q.E.D.*

We now want to show that forming a coalition can be profitable under certain conditions. In order to establish this, we need to make two additional assumptions:

CONDITION 4: $\mu_i(x_i + d_i, d_i)$ *is decreasing in* d_i *for all* $x_i > 0$.

CONDITION 5: $2\mu_i(x_i, 0) \leq \mu_i(2x_i, 0)$.

Alternatively, one may require the following two assumptions, weakening Condition 4 and strengthening Condition 5.

CONDITION 4´: $\mu_i(x_i + d_i, d_i)$ *is non-increasing in* d_i *for all* $x_i > 0$.

CONDITION 5´: $2\mu_i(x_i, 0) < \mu_i(2x_i, 0)$ *for all* $x_i > 0$.

Condition 4 says that the marginal risk concession of a player is decreasing in one's fall-back position. Unlike Assumption 4, Condition 4 is a relatively strong assumption and rules out, for instance, risk-neutral preferences. Condition 4´ relaxes Condition 4 to a weak inequality. Condition 5 says that when the breakdown point is equal to zero, doubling the amount of stake at least doubles the marginal risk concession of a player. Condition 5´ requires that doubling the amount of stake more than doubles the marginal risk concession of a player.

Note that preferences that can be represented by von Neumann-Morgenstern utility functions with constant relative aversion, i.e., $v(x) = x^{\gamma}$ where $0 < \gamma < 1$, satisfy Conditions 4 and 5, while preferences that can be represented by von Neumann-Morgenstern utility functions with constant absolute aversion, i.e., $v(x) = (1 - e^{-x})/(1 - e^{-1})$, satisfy Conditions 4´ and 5´. In Appendix H, we will show Condition 4 is in fact satisfied by a broad class of utility functions that exhibit constant hyperbolic absolute risk aversion (HARA), which include the class of utility functions with constant relative aversion.

The main issue of this paper is whether there are gains from forming a coalition. We will first show that forming a coalition is profitable under the above assumptions.

THEOREM 3.2: *Suppose that either Conditions 4 -5 or Conditions 4´ -5´ are satisfied. A coalition of players with identical preferences will gain as a whole in each market. Formally, if* $(x_c^A, x_c^B, x_{\hat{a}}, x_{\hat{b}})$ *is a simultaneous Nash solution, then* $x_c^A > n_a$ *and* $x_c^B > n_b$.

PROOF: If cable operators have identical preferences, they will split any payoff of the coalition equally. Thus Proposition 3.1 implies that

$$\mu_c(x_c^A + x_c^B, x_c^B) = 2\mu_a\left(\frac{x_c^A + x_c^B}{2}, \frac{x_c^B}{2}\right).$$

But, if Conditions 4 and 5 are satisfied, one has

$$2\mu_a\left(\frac{x_c^A + x_c^B}{2}, \frac{x_c^B}{2}\right) < 2\mu_a\left(\frac{x_c^A}{2}, 0\right) \le \mu_a(x_c^A, 0) \tag{6}$$

since $x_c^B > 0$. Therefore, no $x_c^A \le n_a$ can satisfy equations (1) and (3) in Definition 3.2, for in this case one would have

$$
\begin{aligned}
&\mu_c(x_c^A + x_c^B, x_c^B) \\
&< \mu_a(x_c^A, 0) \\
&\leq \mu_a(n_a, 0) \\
&= \mu_{\hat{a}}(\pi^A - n_a, 0) \\
&\leq \mu_{\hat{a}}(\pi^A - x_c^A, 0),
\end{aligned}
$$

which is absurd. A symmetric argument applies to market *B*, which completes the proof under Conditions 4 and 5.

If Conditions 4´ and 5´ are satisfied instead, the weak and strict inequalities in (6) are exchanged. The proof is the same otherwise. *Q.E.D*.

There are two intuitive explanations as to why forming a coalition is profitable. In fact, the proof of the above proposition is based on these two explanations. Depending on which pair of conditions, 4-5 or 4´-5´, is used, greater emphasis is placed on either of the two explanations.

The first explanation, which is highlighted by Conditions 4 and 5, is as follows: When bargaining in one market, the breakdown point of the coalition is the outcome of the other market. Thus, if bargaining on one frontier breaks down, the coalition still receives some payoff from bargaining on the other frontier. Due to Condition 4, this lowers the coalition's marginal risk concession and thus the coalition can credibly demand a larger share of the pie. This phenomenon may be called the *fall-back position effect*.

The second explanation, which is highlighted by Conditions 4´ and 5´, is as follows: The two members of the coalition share the spoils from each market. Due to Condition 5´, dividing a given payment between two players leads to a lower marginal risk concession than giving the undivided payment to one player. This increases the bargaining power of the coalition. This phenomenon may be called the *risk-sharing effect*.

It is interesting to note that when players with constant relative aversion form a coalition, there is a positive fall-back position effect but zero risk sharing effect, while when players with constant absolute aversion form a coalition, there is a positive risk sharing but zero fall-back position effect.

Theorem 3.2 shows that forming a coalition is profitable. But a profitable coalition may not form if there is no mechanism to divide the gains of the coalition between its members so that each member will gain. If they could write a binding contract regarding the division of the gains, a profitable coalition will always form. This case will be studied in the next subsection.

In the current subsection, we do not allow commitment by the members of a coalition regrading the internal division of a pie. Thus the amount an agent can receive when bargaining alone becomes irrelevant once he decides to join the coalition. Even in this no-commitment case, however, there are some cases where profitable coalitions will actually form. For instance, if $n_a = n_b$, cable operators with identical preferences will both benefit from forming a coalition.

THEOREM 3.3: *Suppose either Conditions 4 -5 or Conditions 4´ -5´ are satisfied. Suppose that two cable operators have identical preferences, two broadcasters have identical preferences, and* $\pi^A = \pi^B$. *Then there exists a simultaneous Nash solution. Furthermore, any simultaneous Nash solution is a bona fide solution to the parallel bargaining problem.*

PROOF: By Proposition 3.2, Conditions 1 and 2 are satisfied. By Condition 4 or 4´, Condition 3 is also satisfied. Thus, by Lemma 3.2, the functions $\tilde{x}_c^A(x_c^B)$ and $\tilde{x}_c^B(x_c^A)$ in the proof of Lemma 3.1 are smooth and increasing. Furthermore, since the cable operators have identical preferences, the broadcasters have identical preferences, and $\pi^A = \pi^B$, the functions $\tilde{x}_c^A(x_c^B)$ and $\tilde{x}_c^B(x_c^A)$ are identical. Therefore, there exists a simultaneous Nash solution $(x_c^A, x_c^B, x_{\hat{a}}, x_{\hat{b}})$ such that $x_c^A = x_c^B$.

Since $x_c^A > n_a$ and $x_c^B > n_b$ by Theorem 3.2 and $n_a = n_b$ by the symmetry of preferences and market sizes, one has

$$S_i^{\sigma}(x_c^A + x_c^B, x_c^B) = \frac{x_c^A + x_c^B}{2} > \frac{n_a + n_b}{2} = n_i \text{ for } i = a, b.$$

Therefore, $(x_c^A, x_c^B, x_{\hat{a}}, x_{\hat{b}})$ is a bona fide solution to the parallel bargaining problem. *Q.E.D.*

In the scenario we studied in this subsection, the members of a coalition split the spoil after it is realized because they cannot make a commitment regarding the split. In this case, it is relatively easy for the players to reach a simultaneous Nash solution once a coalition forms. But it is more difficult to insure that each member of the coalition has an incentive to join a coalition. In the alternative scenario we will study in the next subsection, the members of a coalition can write a binding contract. In this case, it turns out that the opposite is true. It will be more difficult for the players to reach a simultaneous Nash solution (in the sense that establishing its existence requires stronger conditions). But the solution insures that each member gains from joining the coalition.

3.3. *Commitment Solution*

We now consider the solution for the case where the cable operators can write a binding contract when they integrate. The contract between the members of a coalition specifies how they would split the total payoffs in two possible states of nature, one in which bargaining with an outsider, in our example a broadcaster, ends in an agreement and another in which the bargaining breaks down.

Throughout this subsection, we will assume the following:

ASSUMPTION 5: (*Risk Aversion*) *Players prefer the expected value of a gamble to the gamble itself.*

Recall that Assumption 4 introduced in Subsection 2.2 is satisfied for all risk averse players. Thus in this subsection, we do not need Assumption 4 as a separate assumption.

As in Subsection 3.1, denote the agreement and breakdown states by σ and τ, respectively. Let q and $1 - q$ be the probabilities of states by σ and τ, respectively. Note that these probabilities were irrelevant for the no-commitment solution of the previous subsection, for the within-coalition bargaining occurs after either state is realized. If $x_c + d_c$ and d_c are the coalition's payoffs in the agreement and breakdown states, respectively, the contingent pie up for bargaining between the two members of the coalition is, by abuse of notation,

$$\pi = (\pi^{\sigma}, \pi^{\tau}) = (x_c + d_c, d_c) = q \bullet (x_c + d_c) \oplus (1 - q) \bullet d_c .$$

When they bargain over this contingent pie, their respective breakdown points will be the payoffs they expect to receive when they do not join the coalition. Since we are using the Nash solution for any bargaining situation throughout this paper, the fall-back positions of cable operators *a* and *b* will be $n_a = N_a\langle(a, \hat{a}), \pi^A, (0, 0)\rangle$ and $n_b = N_b\langle(b, \hat{b}), \pi^B, (0, 0)\rangle$, respectively.

We will assume that the within-coalition contract is the Nash solution of the contingent-pie bargaining problem $\langle(a, b), (\pi^\sigma, \pi^\tau), (n_a, n_b)\rangle$. Let

$$(y_i, d_i) = q \bullet y_i \oplus (1-q) \bullet d_i = N_i\langle(a, b), (\pi^\sigma, \pi^\tau), (n_a, n_b)\rangle$$

for $i = a, b$. As explained in Subsection 2.3, the Nash solution (y_a, d_a, y_b, d_b) is a pair of contingent shares such that the marginal risk concessions of the two members of the coalition are equalized and such that the allocation of the shares between the two members across the two states is Pareto efficient. That is, the Nash solution satisfies the two equations

$$\text{(7)} \qquad \hat{\mu}_a^\sigma(q \bullet y_a \oplus (1-q) \bullet d_a, n_a) = \hat{\mu}_b^\sigma(q \bullet y_b \oplus (1-q) \bullet d_b, n_b),$$

$$\text{(8)} \qquad m_a(y_a, d_a) = m_b(y_b, d_b)$$

in addition to the two feasibility constraints

$$\text{(9)} \qquad y_a + y_b = x_c + d_c,$$

$$\text{(10)} \qquad d_a + d_b = d_c.$$

In using equation (8) above as a necessary condition for Pareto efficiency, we have used the assumption (Assumption 5) that the members of the coalition are risk averse.

Note here that there was no analogue to equation (8) in the no-commitment case of the previous subsection. Since players could not write a contract, the pie was split according to the Nash solution even in a breakdown state, for otherwise one of the players would have an incentive to renegotiate.

We are particularly interested in the Nash solution of the within-coalition bargaining for the limiting case where q, the probability of the agreement state, approaches 1. In this case, equation (7) above will become

$$\hat{\mu}_a^{\sigma}(1 \bullet y_a \oplus 0 \bullet d_a, n_a) = \hat{\mu}_b^{\sigma}(1 \bullet y_b \oplus 0 \bullet d_b, n_b) .$$

The left hand side can be rewritten in utility terms as

$$\hat{\mu}_a^{\sigma}(1 \bullet y_a \oplus 0 \bullet d_a, n_a) = \frac{v_i(y_a) - v_i(n_a)}{v_i{}'(y_a)} .$$

Note that the expression on right hand side is the same as the marginal risk concession with non-contingent pies introduced in Subsection 2.2. Thus one may write

$$\hat{\mu}_a^{\sigma}(1 \bullet y_a \oplus 0 \bullet d_a, n_a) = \mu_a(y_a, n_a) .$$

Therefore, equation (7) can be replaced by

(11) $$\mu_a(y_a, n_a) = \mu_b(y_b, n_b) .$$

This, together with equation (9) leads to

(12) $$(y_a, y_b) = N\langle (a, b), x_c + d_c, (n_a, n_b) \rangle .$$

Once (y_a, y_b) is determined this way, (d_a, d_b) can be determined from equations (8) and (10).

DEFINITION 3.5: In the case where the members of a coalition can make commitment as to the division of the coalition's share of the pie, the *division scheme* is defined as

$$S(x_c + d_c, d_c) = (y_a, d_a, y_b, d_b),$$

where (y_a, d_a, y_b, d_b) satisfies (12), (8), and (10).

One can combine Definitions 3.1 and 3.5.

PROPOSITION 3.3: *In the case where the members of a coalition can make commitment as to the division of the coalition's share of the pie, the marginal risk concession of the coalition can be written as*

$$\mu_c(x_c + d_c, d_c) = \mu_a(y_a, d_a) + \mu_b(y_b, d_b),$$

where (y_a, y_b) *satisfies (12) and* (d_a, d_b) *satisfies equations (8) and (10).*

We want to show that in the commitment case, there exists a bona fide solution, that is, a simultaneous Nash solution where each member of the coalition gains from joining the coalition. In order to show this, it is necessary that $\mu_c(x_c^A + x_c^B, x_c^B)$ and $\mu_c(x_c^B + x_c^A, x_c^A)$ is well defined outside of the bona fide solution. In particular, it is necessary that $N\langle(a, b), x_c^A + x_c^B, (n_a, n_b)\rangle$ is defined even for the case where $x_c^A + x_c^B < n_a + n_b$.

In order to define $N\langle(a, b), y_a, (n_a, n_b)\rangle$ for the case where $y_a < n_a + n_b$, we imagine the players sharing a loss so that (11) is satisfied, i.e.,

$$\frac{v(y_a) - v(n_a)}{v'(y_a)} = \frac{v(y_b) - v(n_b)}{v'(y_b)},$$

where $0 \le y_a < n_a$ and $0 \le y_b < n_b$. In other words, the marginal risk concession, which is negative in a situation where players have to share a loss, has to be equalized across players. We want to emphasize here that this is only a technical convention. There are no losses at a bona fide solution, whose existence we are going to establish, because the cable operators would not form a coalition if there are losses.

PROPOSITION 3.4: *If commitment is possible regarding the division of the coalition's share of the pie, Condition 1 is satisfied.*

PROOF: If $x_c = 0$, equations (8), (9), and (10) imply $(y_a, y_b) = (d_a, d_b)$ by Assumption 5. Therefore, $\mu_a(y_a, d_a) = \mu_b(y_b, d_b) = 0$ and thus $\mu_c(d_c, d_c) = 0$. *Q.E.D.*

For the commitment case we are analyzing in this subsection, Condition 2, which we needed for Lemma 3.1 may not hold in general. The reason is that as x_c increases, there are in general two effects. First, by Proposition 2.9, both players' payoffs, y_a and y_b, increase. This would increase their marginal risk concessions if (d_a, d_b) remained the same. But the change in (y_a, y_b) also affects the marginal rate of substitution between the agreement and breakdown states. That is,

(d_a, d_b) is affected through equation (8). The direction of this effect on the marginal risk concession of the coalition is in general ambiguous.

THEOREM 3.4: *If commitment is possible regarding the division of the coalition's share of the pie and Conditions 2 and 3 are satisfied, there exists a bona fide solution to the parallel bargaining problem with a one-sided coalition.*

PROOF: That there exists a simultaneous Nash solution follows from Lemma 3.1, Proposition 3.4, and the assumption that Condition 2 is satisfied. With the additional assumption that Condition 3 is satisfied, we can further show that there actually exists a simultaneous Nash solution where each member of the coalition gains from joining the coalition.

Functions $\tilde{x}_c^A(x_c^B)$ and $\tilde{x}_c^B(x_c^A)$ are smooth and non-decreasing by Conditions 2, 3, and Lemma 3.2, and are, respectively, bounded between 0 and π^A and between 0 and π^B. Thus, as can be seen from Figure 2, there has to exist a simultaneous Nash solution such that $x_c^A > n_a$ and $x_c^B > n_b$ if one could establish $\tilde{x}_c^A(n_b) > n_a$ and $\tilde{x}_c^B(n_a) > n_b$.

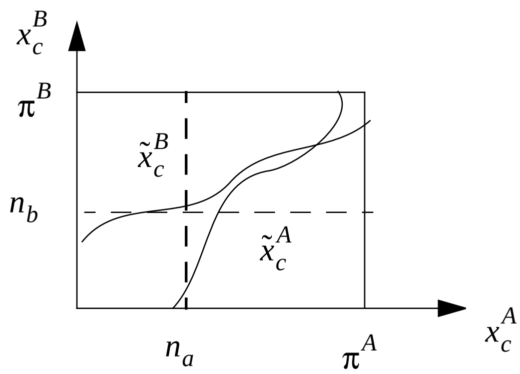

FIGURE 2

Thus we have only to show that

(13) $\mu_c(n_a + n_b, n_b) < \mu_a(n_a, 0)$.

By Proposition 3.3,

$$\mu_c(n_a + n_b, n_b) = \mu_a(y_a, d_a) + \mu_b(y_b, d_b)$$

where (y_a, d_a, y_b, d_b) satisfy

(14) $\mu_a(y_a, n_a) = \mu_b(y_b, n_b)$,

(15) $m_a(y_a, d_a) = m_b(y_b, d_b)$,

(16) $y_a + y_b = n_a + n_b$,

(17) $d_a + d_b = n_b$.

Note that equations (14) and (16) imply that $y_a = n_a$ and $y_b = n_b$. Thus

(18) $\mu_c(n_a + n_b, n_b) = \mu_a(n_a, d_a) + \mu_b(n_b, d_b)$.

Hence, equation (13) is satisfied if

(19) $$\mu_a(n_a, d_a) + \mu_b(n_b, d_b) < \mu_a(n_a, 0),$$

which is eqiuvalent to

(20) $$\mu_b(n_b, d_b) < \mu_a(n_a, 0) - \mu_a(n_a, d_a),$$

or in utility form,

(21) $$\frac{v_b(n_b) - v_b(d_b)}{v_b'(n_b)} < \frac{v_a(d_a)}{v_a'(n_a)}.$$

But equation (15) implies that equation (21) is equivalent to

(22) $$\frac{v_b(n_b) - v_b(d_b)}{v_b'(d_b)} < \frac{v_a(d_a)}{v_a'(d_a)}.$$

But, by the concavity of v_b, equation (17), and the concavity of v_a, one has

(23) $$\frac{v_b(n_b) - v_b(d_b)}{v_b'(d_b)} < (n_b - d_b) = d_a < \frac{v_a(d_a)}{v_a'(d_a)}.$$ Q.E.D.

Theorem 3.2, which established the profitability of a coalition for the no-commitment case, relied on two effects, the fall-back position effect and the risk-sharing effect. The proof of the above Theorem 3.4 reveals that similar effects are at work for the commitment case. The use of Condition 3 in establishing the monotonicity of the functions $\tilde{x}_c^A(x_c^B)$ and $\tilde{x}_c^B(x_c^A)$ indicates the presence of the fall-back position effect. For Theorem 3.2, the risk-sharing effect worked through Condition 5´. For Theorem 3.4, however, the risk-effect works through risk aversion as can be seen from the last part of the proof.

Overall, the ability to write a binding contract increases the opportunity to gain from forming a coalition. The coalition becomes a more effective bargainer than an individual if certain conditions are met. Conditions 2 and 3 in Theorem 3.4 are aggregation conditions that require that the coalition's aggregate preferences exhibit certain desirable properties.

4 . CONCLUSION

In this paper, we have provided theoretical explanations for bargaining power due to integration across local markets. We extended the Nash solution to the case of parallel bargaining to illustrate why players might gain from integration in two alternative scenarios: one in which players who form a coalition cannot write a binding contract, and the other in which players can write a binding contract. We showed that the integration can increase bargaining power under certain conditions.

From the policy standpoint, the results support the view that across-local-market integration increases market power. Integration leads to a redistribution of some of the gains from cooperation within the local market from the unintegrated to the integrated players. Since, however, rational players will always exhaust all possible gains from cooperation within the local market, there is no justification, within our model, for restricting the national size of an MSO in the cable television industry or restricting the size of a theater chain in the movie industry. Even though integration may increase their market power, it does not affect aggregate welfare. An interesting open problem is to find a model where policy makers should be concerned about the MSOs' and theater chains' market power on efficiency grounds.

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APPENDIX A: Constant Relative Risk Aversion implies Assumption 4

If $v(x) = x^{\gamma}$, one has

$$\mu(x+d, d) = \frac{(x+d)^{\gamma} - d^{\gamma}}{f(x+d)^{\gamma-1}}$$

and thus

$$\frac{\partial\mu(x+d, d)}{\partial x} = \frac{(\gamma(x+d)^{\gamma-1})^2 - \gamma(\gamma-1)(x+d)^{\gamma-2}[(x+d)^{\gamma} - d^{\gamma}]}{[\gamma(x+d)^{\gamma-1}]^2}.$$

It is easy to see that for all $\gamma \le 1$, the numerator, and hence the derivative, is positive. The following manipulation shows that the numerator is also positive for all $\gamma > 1$:

$$\begin{aligned}&\{\gamma(x+d)^{\gamma-1}\}^2 - \gamma(\gamma-1)(x+d)^{\gamma-2}\{(x+d)^{\gamma} - d^{\gamma}\}\\ &= \gamma\{\gamma(x+d)^{2\gamma-2} - (\gamma-1)(x+d)^{2\gamma-2} + (\gamma-1)(x+d)^{\gamma-2}d^{\gamma}\}\\ &= \gamma\{(x+d)^{2\gamma-2} + (\gamma-1)(x+d)^{\gamma-2}d^{\gamma}\} > 0.\end{aligned}$$

APPENDIX B: Proof of Proposition 2.12

Define a function $z_i^{\tau}(z_i^{\sigma})$ such that $(z_i^{\sigma}, z_i^{\tau}(z_i^{\sigma})) \in C$. To derive $\mu_i^{\sigma}(y_i, d_i)$, we will use the following proposition:

PROPOSITION A.1: For any given $y_i \succsim_i z_i \succsim_i d_i$, where $y_i \succ_i d_i$, one has

$$z_i^{\sigma} = s_i^{\sigma}\left(\frac{qv_i(z_i^{\sigma}) + (1-q)v_i(z_i^{\tau}(z_i^{\sigma})) - v_i(d_i)}{qv_i(y_i^{\sigma}) + (1-q)v_i(y_i^{\tau}) - v_i(d_i)}, y_i, d_i\right).$$

PROOF: There exists some p such that

$$q \bullet z_i^\sigma \oplus (1-q) \bullet z_i^\tau(z_i^\sigma) \sim p \bullet \{q \bullet y_i^\sigma \oplus (1-q) \bullet y_i^\tau\} \oplus (1-p) \bullet d_i.$$

Rewriting this expression using the utility representation in Proposition 2.1, we obtain

$$qv_i(z_i^\sigma) + (1-q)v_i(z_i^\tau(z_i^\sigma)) = p\{qv_i(y_i^\sigma) + (1-q)v_i(y_i^\tau)\} + (1-p)v_i(d_i),$$

i.e.,

$$p = \frac{qv_i(z_i^\sigma) + (1-q)v_i(z_i^\tau(z_i^\sigma)) - v_i(d_i)}{qv_i(y_i^\sigma) + (1-q)v_i(y_i^\tau) - v_i(d_i)}.$$

Thus, by Definition 2.8, one obtains the proposition. *Q.E.D.*

PROOF OF PROPOSITION 2.12: From Definitions 2.9, 2.10, and the L'Hospital's rule, we have

$$\mu_i^\sigma(y_i, d_i) = \frac{ds_i^\sigma(1, y_i, d_i)}{dp}.$$

Totally differentiating both sides of the equality in Proposition A.1 with respect to z_i^σ yields

$$1 = \frac{ds_i^\sigma}{dp}\left(\frac{qv_i(z_i^\sigma) + (1-q)v_i(z_i^\tau(z_i^\sigma)) - v_i(d_i)}{qv_i(y_i^\sigma) + (1-q)v_i(y_i^\tau) - v_i(d_i)}, y_i, d_i\right)\frac{qv_i'(z_i^\sigma) + (1-q)v_i'(z_i^\tau(z_i^\sigma))\frac{dz_i^\tau}{dz_i^\sigma}}{qv_i(y_i^\sigma) + (1-q)v_i(y_i^\tau) - v_i(d_i)}.$$

Setting $z_i = y_i$ and rewriting gives the desired expression. $\mu_i^\tau(y_i, d_i)$ can be derived similarly.

Q.E.D.

APPENDIX C: Proof of Proposition 2.13

From Proposition 2.12 and Definition 2.12, one has

$$\mu_i(y_i, d_i) = \hat{\mu}_i^{\sigma}(y_i, d_i)\frac{1}{q + (1-q)\dfrac{v_i'(y_i^{\tau})}{v_i'(y_i^{\sigma})}\cdot\dfrac{dy_i^{\tau}}{dy_i^{\sigma}}}\left(1, \frac{dy_i^{\tau}}{dy_i^{\sigma}}\right),$$

$$\mu_j(\pi - y_i, d_j) = \hat{\mu}_j^{\sigma}(\pi - y_i, d_j)\frac{1}{q + (1-q)\dfrac{v_j'(\pi^{\tau} - y_i^{\tau})}{v_j'(\pi^{\sigma} - y_i^{\sigma})}\cdot\dfrac{dy_i^{\tau}}{dy_i^{\sigma}}}\left(1, \frac{dy_i^{\tau}}{dy_i^{\sigma}}\right).$$

Along the *C* curve, one has $\dfrac{v_i'(y_i^{\tau})}{v_i'(y_i^{\sigma})} = \dfrac{v_j'(\pi^{\tau} - y_i^{\tau})}{v_j'(\pi^{\sigma} - y_i^{\sigma})}$, which proves the proposition.

APPENDIX D: Proof of Proposition 2.14

Differentiating $\hat{\mu}_i^{\sigma}(y_i, d_i)$ with respect to y_i^{σ}, gives

$$\frac{d\hat{\mu}_i^{\sigma}(y_i, d_i)}{dy_i^{\sigma}}$$

$$= \frac{\left\{qv_i'(y_i^{\sigma}) + (1-q)v_i'(y_i^{\tau})\dfrac{v_i'(y_i^{\tau})}{v_i'(y_i^{\sigma})}\right\}v_i'(y_i^{\sigma}) - v_i''(y_i^{\sigma})\{qv_i(y_i^{\sigma}) + (1-q)v_i(y_i^{\tau}) - v_i(d_i)\}}{\{v_i'(y_i^{\sigma})\}^2},$$

which is positive because $v_i''(y_i^{\sigma}) < 0$.

APPENDIX E: Proof of Proposition 2.15

Using Proposition 2.13, we have only to show that there exist a unique y_i^{σ} satisfying

$$\hat{\mu}_i^{\sigma}(y_i, d_i) = \hat{\mu}_j^{\sigma}(\pi - y_i, d_j) .$$

From Proposition 2.14, $\hat{\mu}_i^{\sigma}(y_i, d_i)$ is increasing in y_i. By Proposition 2.11, as y_i increases along the *C* curve, player *j* receives less in both states, and thus $\pi - y_i$ decreases. If $y_i \sim_i d_i$, the left hand side of the above equation is zero, and if $\pi - y_i \sim_i d_j$, the right hand side is equal to zero. Since the left hand side is continuously increasing in y_i and the right hand side continuously decreasing in y_i, there exist a unique solution.

APPENDIX F: Rubinstein Solution for a Contingent-Pie Bargaining Problem

DEFINITION A.1: The Rubinstein solution to a contingent-pie bargaining problem $\langle (i, j), \pi, (d_i, d_j) \rangle$ is a vector of payoffs, $((y_i, \hat{y}_j), (\hat{y}_i, y_j))$, where $y_i, \hat{y}_i \in C$ such that

(A.1) $\hat{y}_i = s_i(p, y_i, d_i)$,

(A.2) $\hat{y}_j = s(p, y_j, d_j)$.

The Rubinstein solution consists of two pairs of payoff vectors. The first pair is the outcome that is realized when player *i* is the proposer in an alternating offer model and the second pair is the outcome that is realized when player j is the proposer. Conditions (A.1) and (A.2) ensure that each player, when he is a responder, is indifferent between accepting an offer and rejecting it. If he rejects, he can become a proposer but he also risks a breakdown. Here one can easily see that the Rubinstein solution is the equilibrium outcome of a strategic bargaining model similar to the one in Rubinstein (1982).

PROPOSITION A.2: *The Rubinstein solution for a contingent pie equalizes the players' risk concessions.*

PROOF: From conditions (A.1) and (A.2) in the definition of the Rubinstein solution and the fact that we work with elements of C only, it follows that

$$y_i + s_j(p, y_j, d_j) = \pi,$$

$$s_i(p, y_i, d_i) + \hat{y}_j = \pi.$$

Subtracting the second equation from the first one yields

$$c_i(p, y_i, d_i) = c_j(p, y_j, d_i). \qquad \textit{Q.E.D.}$$

APPENDIX G: Nash Solution in Utilities with a Contingent Pie

In this appendix, we will show that the Nash solution for a contingent pie we defined in terms of preferences is equivalent to the Nash solution defined in terms of the von Neumann-Morgenstern utilities. Consider the bargaining problem $\langle (i, j), \pi, (d_i, d_j) \rangle$ as defined in the text. Assume that players' preferences can be represented by concave von Neumann-Morgenstern utility functions. Then the Nash solution can be found by the following optimization problem:

$$max_{x_i^\sigma, x_i^\tau}\{qv_i(x_i^\sigma + d_i) + (1-q)v_i(x_i^\tau + d_i)) - v_i(d_i)\} \cdot$$
$$\left\{qv_j(\pi^\sigma - x_i^\sigma - d_i) + (1-q)v_j(\pi^\tau - x_i^\tau - d_i) - v_j(d_j)\right\}.$$

The first order conditions for this maximization problem can be rearranged to yield the following two equations:

$$\frac{qv_i(x_i^\sigma + d_i) + (1-q)v_i(x_i^\tau + d_i) - v_i(d_i)}{v_i'(x_i^\sigma + d_i)} = \frac{qv_j(\pi^\sigma - x_i^\sigma - d_i) + (1-q)v_j(\pi^\tau - x_i^\tau - d_i) - v_j(d_j)}{v_j'(\pi^\sigma - x_i^\sigma - d_i)},$$

$$\frac{v'_i(x_i^\sigma + d_i)}{v'_i(x_i^\tau + d_i)} = \frac{v'_j(\pi^\sigma - x_i^\sigma - d_i)}{v'_j(\pi^\tau - x_i^\tau - d_i)}.$$

Note that the second equation equalizes the marginal rates of substitution between different states of nature across agents. Thus the second equation ensures that the outcome is an element of the *PE* curve. The first equation defines an outcome at which $\hat{\mu}_i^\sigma(y_i, d_i) = \hat{\mu}_j^\sigma(\pi - y_i, d_j)$.

APPENDIX H: Example of Preferences Satisfying Condition 4

We will show that Condition 4 is satisfied by all preferences that can be represented by von Neumann-Morgenstern utility functions with constant hyperbolic absolute risk aversion (HARA), i.e.,

$$v(x) = \frac{1-\gamma}{\gamma}\left(\frac{ax}{1-\gamma}+b\right)^{\gamma} \text{ where } -\infty < \gamma < 1 .$$

Here we have not normalized the function $v(x)$ so that $v(0) = 0$ and $v(1) = 1$ as we did in the text. The limiting case where $\gamma \to 0$ corresponds to the logarithmic utility function, i.e., $v(x) = \ln(x+1)$. The restriction $\gamma < 1$ ensure that agents are risk averse. This class of utility function is broad and, for example, includes all utility functions with constant relative risk aversion, i.e. $v(x) = x^{\gamma}$, where $0 < \gamma < 1$.

We want to show $\frac{\partial\mu(x+d, d)}{\partial d} < 0$ for $x > 0$. One has

$$\mu(x+d, d) = \frac{\frac{1-\gamma}{\gamma}\left\{\left(\frac{a(x+d)}{1-\gamma}+b\right)^{\gamma} - \left(\frac{ad}{1-\gamma}+b\right)^{\gamma}\right\}}{a\left(\frac{a(x+d)}{1-\gamma}+b\right)^{\gamma-1}}$$

$$= \frac{1-\gamma}{a\gamma}\left\{\left(\frac{a(x+d)}{1-\gamma}+b\right)^{\gamma} - \left(\frac{ad}{1-\gamma}+b\right)^{\gamma}\right\}\left(\frac{a(x+d)}{1-\gamma}+b\right)^{1-\gamma}$$

$$= \frac{1-\gamma}{a\gamma}\left\{\left(\frac{a(x+d)}{1-\gamma}+b\right) - \left(\frac{ad}{1-\gamma}+b\right)^{\gamma}\left(\frac{a(x+d)}{1-\gamma}+b\right)^{1-\gamma}\right\},$$

and thus

$$\frac{\partial\mu(x+d, d)}{\partial d} = \frac{1-\gamma}{a\gamma}\left\{\frac{a}{1-\gamma} - a\left(\frac{a(x+d)}{1-\gamma}+b\right)^{-\gamma}\left(\frac{ad}{1-\gamma}+b\right)^{\gamma} - \frac{\gamma a}{1-\gamma}\left(\frac{a(x+d)}{1-\gamma}+b\right)^{1-\gamma}\left(\frac{ad}{1-\gamma}+b\right)^{\gamma-1}\right\}.$$

First consider the case where $0 < \gamma < 1$. In this case, we have $\frac{\partial\mu(x+d,d)}{\partial d} < 0$ if and only if

$$1-(1-\gamma)\left[\frac{\frac{ad}{1-\gamma}+b}{\frac{a(x+d)}{1-\gamma}+b}\right]^{\gamma}-\gamma\left[\frac{\frac{ad}{1-\gamma}+b}{\frac{a(x+d)}{1-\gamma}+b}\right]^{\gamma-1}<0.$$

This is equivalent to

$$\left(\frac{a(x+d)}{1-\gamma}+b\right)^{\gamma}+\gamma\left(\frac{ad}{1-\gamma}+b\right)^{\gamma}<\left(\frac{ad}{1-\gamma}+b\right)^{\gamma}+\gamma\left(\frac{ad}{1-\gamma}+b\right)^{\gamma-1}\left(\frac{a(x+d)}{1-\gamma}+b\right).$$

Note that for $x = 0$ the left hand side is equal to the right hand side. Furthermore,

$$\frac{\partial LHS}{\partial x} = \gamma\left(\frac{a(x+d)}{1-\gamma}+b\right)^{\gamma-1}\frac{a}{1-\gamma},$$

while

$$\frac{\partial RHS}{\partial x} = \gamma\left(\frac{ad}{1-\gamma}+b\right)^{\gamma-1}\frac{a}{1-\gamma}.$$

Thus we have $\frac{\partial LHS}{\partial x} < \frac{\partial RHS}{\partial x}$, which establishes that $\frac{\partial\mu(x+d,d)}{\partial d} < 0$ for $x > 0$.

Similarly, one can show that Condition 4 is satisfied for the cases where $\gamma = 0$ and $\gamma < 0$.

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