

**EXERCISE 5: Answers**

This exercise considers the estimation of the marginal propensity to consume and whether consumption decisions are made with reference to current income or permanent income as proposed by Friedman. This parameter is important because in the simplest Keynesian model the multiplier for government expenditures is  $1/(1 - \beta)$  where  $\beta$  is the marginal propensity to consume. The data set used is Koyck.dat which is available on the course website as are sample GAUSS code for OLS and IV estimation.

1. Duesenberry considered the following model for aggregate consumption:

$$C_t = \alpha + \beta Y_t + u_t$$

where  $u_t$  is an unknown disturbance term.

- Run an OLS regression based on this specification and report  $\hat{\alpha}$  and  $\hat{\beta}$ .  $\hat{\alpha} = -7.43359$  and  $\hat{\beta} = 0.953052$ .
  - Making explicit the assumptions you are making, what are the properties of your estimates  $\hat{\alpha}$  and  $\hat{\beta}$ . *Under assumptions (i)-(v) the OLS estimates are unbiased, BLUE, and consistent. If we add normality (vi) then they are MLE and BUE.*
  - Discuss whether  $\beta > 1$  is an economically sensible possibility. What are the implications for the multiplier if  $\beta$  is less than but close to one? *It would seem impossible to sustain MPC greater than one since it implies long-term continued dissaving. The multiplier in a simple Keynesian model is  $1/(1 - \beta)$ , which will become large as  $\beta$  approaches one.*
  - Test the null hypothesis that the marginal propensity to consume is one. In view of your answer to (c) should this be a one or two-sided test? *This is a one-sided test since greater than one is not economically feasible. We consider  $(\hat{\beta} - \beta_0)/s.e.(\hat{\beta}) = (0.953052 - 1)/0.0137655 = -3.4106$  which should be a draw from the  $t_{35}$  under the null. But this is far into the left-hand tail and rejects the null for any likely choice of size  $\alpha$ .*
2. Friedman considered a variation of the same equation:

$$C_t = \alpha + \beta Y_t^p + u_t$$

where  $Y_t^p$  is permanent income or the discounted present value of all current and future expected streams of income. He modeled this expectation as a geometrically declining weighted average of current and past incomes:

$$Y_t^p = (1 - \lambda)[Y_t + \lambda Y_{t-1} + \lambda^2 Y_{t-2} + \lambda^3 Y_{t-3} + \dots]$$

The multiplication factor  $(1 - \lambda)$  ensures that the level of  $Y_t^p$  is similar to  $Y_t$ .

- a. Show how the model can be transformed to obtain the Koyck transformation

$$\begin{aligned} C_t &= (1 - \lambda)\alpha + \beta(1 - \lambda)Y_t + \lambda C_{t-1} + (u_t - \lambda u_{t-1}) \\ &= \alpha^* + \beta^* Y_t + \lambda C_{t-1} + u_t^* \end{aligned}$$

*If we consider  $C_{t-1} = \alpha + \beta Y_{t-1}^p + u_{t-1}$  and  $Y_{t-1}^p = (1 - \lambda)[Y_{t-1} + \lambda Y_{t-2} + \lambda^2 Y_{t-3} + \lambda^3 Y_{t-4} + \dots]$  and form*

$$C_t - \lambda C_{t-1} = \alpha(1 - \lambda) + \beta(Y_t^p - \lambda Y_{t-1}^p) + (u_t - \lambda u_{t-1})$$

*Note that  $Y_t^p - \lambda Y_{t-1}^p = (1 - \lambda)Y_t$  whereupon we can collect terms on the right to obtain the answer.*

- b. Estimate this equation by least squares and report your results. What assumptions are you making concerning the disturbances for these estimates and associated inferences to

be appropriate?  $\hat{\alpha}^* = -3.15661$ ,  $\hat{\beta}^* = 0.594962$ , and  $\hat{\lambda} = 0.373229$ . Since we have a dependent variable on the RHS we have to consider stochastic regressors. We assume that the implied error term is at least uncorrelated with the regressors, and for inferences need  $(u_t^*, x_t)$  jointly i.i.d.,  $E(u_t^*|x_t) = 0$ , and  $E(u_t^{*2}|x_t) = \sigma^2$ .

- c. Test the null hypothesis that  $\lambda = 0$ . Should this be a one- or two-sided test? This should be a one-sided test since we will likely give positive weight to the previous income in forming our estimate of permanent income. Consider  $(\hat{\lambda} - \lambda)/s.e.(\hat{\lambda}) = (0.373229 - 0)/0.118626 = 3.14627$ , which should be a draw from the  $t_{35}$  under the null. Since the realization is well into the RHS tail we reject for any likely choice of size.
  - d. Test the hypothesis that  $\beta^*$  in this model is one. How could you estimate  $\beta$ ? Note that a null that  $\beta$  is zero also implies that  $\beta^*$  is also zero and visa-versa. Consider  $(\hat{\beta}^* - \beta_0^*)/s.e.(\hat{\beta}^*) = (0.594962 - 1)/0.114476 = -3.5382$  and we continue to reject. We can estimate  $\hat{\beta} = \hat{\beta}^*/(1 - \hat{\lambda}) = .94925$  which is very close to the original estimate.
  - e. Calculate a 95% confidence interval for  $\hat{\beta}^*$ . What is the relationship of this interval to the hypothesis test. From the  $t$ -tables  $t_{35, .025} = 2.03$  thus our 95% interval for  $\hat{\beta}^*$  is  $0.594962 \pm 0.114476 \times 2.03$ . This interval is the set of null values which will not be reject with  $\alpha = .05$ .
3. Suppose the original untransformed disturbances have ideal properties and all the current and past values of  $Y_t$  are strictly exogenous.
- a. Show that  $C_{t-1}$  and  $u_t^*$  are correlated. What does this imply about the least squares estimates obtained in (2)? From above  $C_{t-1} = \alpha + \beta Y_{t-1}^p + u_{t-1}$  and  $u_t^* = u_t - \lambda u_{t-1}$  and the two are correlated since they share the common element  $u_{t-1}$  and  $Y_t^p$  is strictly exogenous. This means the OLS estimates in (2) will be biased and inconsistent.
  - b. Show that  $Y_{t-1}$  is a proper instrument for this case. That is, show that  $Y_{t-1}$  is uncorrelated with  $u_t^*$  but correlated with  $C_t$ . If we consider all the  $Y_t$  to be strictly exogenous then  $Y_{t-1}$  will be uncorrelated with both  $u_{t-1}$  and  $u_t$ . It will be correlated with  $C_{t-1}$ , however since it appears in  $Y_{t-1}^p$ .
  - c. Estimate the model by instrumental variables and report your results. What assumptions are you making for this approach and associated inferences to be appropriate?  $\bar{\alpha}^* = -7.10385$ ,  $\bar{\beta}^* = 0.925445$ , and  $\bar{\lambda} = 0.0287743$ . Note that the implied value of  $\bar{\beta}$  is close to the original estimate. We assume that  $(u_t^*, x_t, z_t)$  jointly i.i.d.,  $E(u_t^*|z_t) = 0$ ,  $E(u_t^{*2}|z_t) = \sigma^2$ , and  $E(z_t x_{t'}) = M$  nonsingular.
  - d. Test the null hypothesis that  $\lambda = 0$ . Also test the hypothesis that  $\beta^* = 1$ . Consider  $(\bar{\lambda} - \lambda)/s.e.(\bar{\lambda}) = (0.02877 - 0)/0.1928 = .14922$  which should, asymptotically, be a draw from a standard normal. This is in the middle of the distribution and will not reject for any likely choice of size.
  - e. Test whether or not  $C_{t-1}$  is correlated with  $u_t^*$ . We consider  $(\hat{\beta}^* - \bar{\beta}^*)/(\text{var}(\bar{\beta}^*) - \text{var}(\hat{\beta}^*))^{1/2} = (0.925445 - 0.594962)/((0.18552)^2 - (0.114476)^2)^{1/2} = 6.9543$ , which should be asymptotically standard normal under the null. We clearly reject the null.