

EXERCISE 3

The following table contains the ACT scores and the GPA (grade point average) for 8 college students. Grade point average is based on a four-point scale and had been rounded to one digit after the decimal. Perform the following calculations by hand and show all your work. Understanding these details is the point of the exercise.

Student	GPA	ACT
1	2.8	21
2	3.4	24
3	3.0	26
4	3.5	27
5	3.6	29
6	3.0	25
7	2.7	25
8	3.7	30

1. Estimate the relationship between GPA and ACT using OLS. That is, obtain the intercept and slope estimates in the equation

$$GPA_i = \beta_0 + \beta_1 ACT_i + u_i.$$

Comment on the direction of the relationship. Does the intercept have a useful interpretation here? Explain. How much higher is the GPA predicted to be, if the ACT score is increased by 5 points?

2. Compute the fitted values and residuals for each observation and verify that the residuals (approximately) sum to zero. Calculate the estimated variance of $\hat{\beta}_1$ and test the null hypothesis that $\beta_1 = 10$.
3. What is the predicted value of GPA when ACT=25? Compare to realizations in the sample and explain why they would be similar or different.
4. Making explicit the assumptions you are making regarding ACT_i and u_i discuss the properties of your estimates (mean, variance, and distribution, if possible).